Investment Decisions and Capital Accumulation: Firm-Level Evidence from Brazil

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November 29, 2023

Abstract

Using firm-level data from a Brazilian industrial survey, I document a set of empirical facts regarding capital stock accumulation patterns and investment decisions. Finding evidence largely in favor of micro-level lumpiness of investment as it was found for American firms, I document that there are particularities in the behavior of Brazilian firms. First, I document that the distribution of the growth rate of capital is more dispersed, with "fatter" tails. Second, I show that episodes of capital expansion and destruction are more intense. Third, I compute statistical measures related to the investment rate distribution. These measures show investment at the firm- level seems to be even lumpier in Brazil, with firms investing less on average, while experiencing more episodes of investment spikes and periods of inaction. Fourth, I show that factors of production are highly concentrated. Finally, using a model that encompasses heterogenous firms facing both convex and non-convex capital adjustment costs, I show that these firm-level facts have aggregate implications, in particular inducing higher volatility to aggregate investment and aggregate output than in an economy with lower levels of lumpiness.

1 Introduction

Understanding investment has long been a fundamental question for macroeconomists, as it plays a fundamental role both in the long-run and in the short run. First, and most importantly, investment is the link between the present and the future in an economy, as it allows to accumulate capital which is the key production factor to induce growth in the long-run, even though productivity growth is the main driver. Second, investment is the most variable component of output and, as such, understanding what generates this variation is of key importance to understand the business cycle.

Even though this is a primarily macroeconomic question, economists, over the last 20 years, have increasingly turned themselves to microeconomic data in order to better understand aggregate dynamics. The failure of the first modeling attempts and the accumulation of new data, econometric methods and theoretical models have been the main drivers of this process, which has been somewhat successful in providing useful insights. Unfortunately, however, such progress has been mainly focused on developed economies, in particular the United States economy.

In this paper, I take advantage of a comprehensive dataset originated from a Brazilian industrial survey to think about what is the role of micro lumpiness, here understood as periods of inaction followed by bursts of activity, in firm investment behavior to understand the business cycle of a developing country. I document four main empirical particularities.

First, I document that the distribution of the growth rate of capital is more dispersed amongst Brazilian firms, with "fatter" tails. Second, I show that episodes of capital expansion and destruction are more intense. Third, I compute statistical measures related to the investment rate distribution. These measures show investment at the firm-level seems to be even lumpier in Brazil, with firms investing less on average, while experiencing more episodes of investment spikes and periods of inaction. Fourth, I show that factors of production are highly concentrated, even more than in US.

To make sense of these empirical observations, I solve and calibrate a model that incorporates irregular investment behavior at the micro level into an otherwise RBC model with heterogenous firms and a consumer that exhibits preferences with habit formation over consumption streams. Even though my calibration is still far from where it should be, the results seem to be in line with Winberry (2021), the closest predecessor to this paper.

Related Literature. — This paper contributes to three main strands of the literature. First, it builds on a long-lasting literature that tries to understand which model of firm investment behavior is more adequate to understand the data. The seminal work by Jorgenson (1963, 1972), Tobin (1968, 1969), Hayashi (1982) focused on the role convex adjustment costs while more recent work by Abel and Eberly (1994, 1996), Dixit (1997), Caballero and

Engel (1993, 1999), Bertola and Caballero (1994), Caballero, Engel and Haltiwanger (1995) emphasized the theoretical importance of non-convex adjustment costs. My results suggest that a combination of both types of adjustment costs are relevant to understand the empirical patterns observed in the data for Brazilian firms, such as suggested in Cooper and Haltinwanger (1999, 2006).

Second, this paper contributes to a more recent literature gathering stylized facts about firm and/or establishment-level investment decisions. In particular, my results connect facts about the growth rates of capital, as in Doms and Dunne (1998) and Carlsson and Láseen (2005), to facts about the investment rate distribution, as in Zwick and Mahon (2017), using unique dataset for Brazilian firms. This will allow to explore how additional sources of heterogeneity and frictions at the firm-level matter to understand aggregate dynamics of investment, which leads to my last point.

Finally, this paper contributes to a large literature that is focused on understanding how irregular decisions at the extensive margin of investment impacts macroeconomic dynamics. This literature has largely been divided in two groups. On one side, we have the contributions of Thomas (2002), Khan and Thomas (2003, 2008), which point out that, in general equilibrium, large surges in investment forecasted by the behavior of firms in partial equilibrium in the aftermath of a positive TFP shock are undone by a desire to smooth consumption which pushes interest rates to rise and counteracts the effect of non-convex adjustment costs.

On the other side, despite some early criticism by Gouriou and Kashyap (2007), we have the recent contributions by Koby and Wolf (2020) and Winberry (2021) which suggest that these results do not hold. The former focuses on the price-elasticity of aggregate investment, arguing that most of these early models imply far too high price-elasticities which seem to be at odds with quasi-experimental evidence from firm-level responses to tax changes. The latter, the closest predecessor to this paper, shows that, once we find a way to match the dynamics of real interest rates as in the data – which also requires breaking this extreme sensitivity –, we also obtain that micro-level lumpiness is relevant to understand aggregate dynamics. As such, this paper contributes to this long-standing debate by providing empirical evidence that supports the more recent papers and by extending the analysis to a relevant emerging market economy, which might eventually allow us to think about policy experiments that are relevant for that environment.

Organization. — This article is organized as follows. In the next section, I present the dataset used in this article, detailing its collection, the selection of my final sample, the definition of the most important variables and the steps needed to construct the capital stock series for each firm. In the third section, I analyze firm-level capital accumulation patterns. The fourth section presents the model and the quantitative results. Finally, the fifth session concludes and outlines the future steps.

2 Data

This paper analyzes data from the *Pesquisa Industrial Anual*[1] (PIA), a confidential dataset maintained at IBGE[2]. This survey has the goal of identifying the structural characteristics of the Brazilian industrial sector, its changes through time and of providing annual information about the volume and value of sales in each sector, which in turn will be used to construct the Brazilian National Accounts.

The first PIA survey was released in 1967, undergoing several methodological revisions since then, with the last one taking place in 1996. As such, the original dataset in this project only covers the period from 1996 until 2015, which was the last available year at the time of the analysis at the IBGE Data Center.

Collection

IBGE uses two different questionnaires — a "complete" model and a "simplified" model — to construct its survey. Both questionnaires gather information from six major areas (labor, revenue, costs and expenditures, acquisitions and write-offs of assets and sectoral information), but the first one allows for a more detailed description of firms. The complete model is filed by all firms with more than 30 employees and/or all firms with revenue of more than a threshold value[3] in the year before the reference year of the survey[4], while the simplified version is filled by a random sample of firms with less than 30 employees. As such, when taking into consideration the population of firms in Brazil, the PIA sample tends to overrepresent larger firms.

Sample Selection

The ultimate goal of this paper is to analyze how firms make their investment decisions and what are the consequences of such decisions to their capital stock accumulation. To do so, I apply a combination of the perpetual inventory method and an estimation of sectoral depreciation rates, which requires that we have a balanced panel of firms for the period between 1996 and 2015. To reduce results' dependence on initial conditions for the capital

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[1] Annual Manufacturing Survey – Direct translation from Portuguese.

[2] Instituto Brasileiro de Geografia e Estatística – Brazilian Bureau of Geography and Statistics, in a direct translation from Portuguese.

[3] This value changes from year to year. In 2015, it was R$ 12.8 millions.

[4] The completion of the questionnaire is mandatory for these firms.

stock series – more details on this in the next section –, I restrict the empirical analysis to the period between 2000 and 2015.

As such, the final dataset contains 5191 firms across a total of 107 sectors, out of a total of 144666 firms that appear at least once in the original sample between 1996 and 2015 also across a total of 107 sectors. Even though the coverage in terms of number of firms is small, the establishments present in the final dataset account for a large share of investment, labor and production and revenue and are quite large, as one can observe from Table 1.

Table 1: Sample Coverage by Year

Year	Employment Coverage (%)	Revenue Coverage (%)	Production Coverage (%)	Investment Coverage (%)	Average Employment
2000	41.7%	51.9%	55.6%	47.3%	296.1
2001	41.5%	51.7%	54.3%	53.8%	301.8
2002	41.5%	52.4%	55.4%	60.2%	315.0
2003	41.3%	53.3%	57.3%	55.8%	326.3
2004	41.1%	53.8%	57.8%	61.0%	358.3
2005	40.3%	54.5%	58.7%	59.1%	366.6
2006	39.7%	54.4%	58.4%	67.3%	380.0
2007	40.4%	55.8%	58.6%	59.2%	410.1
2008	40.5%	55.9%	59.1%	58.9%	425.6
2009	40.7%	55.2%	58.9%	69.4%	430.6
2010	40.5%	55.7%	60.1%	65.6%	459.2
2011	40.2%	55.5%	60.3%	63.3%	475.1
2012	39.9%	54.7%	58.7%	60.0%	483.4
2013	40.1%	55.4%	58.9%	62.5%	490.6
2014	40.2%	55.1%	57.4%	56.6%	482.8
2015	40.9%	55.2%	57.2%	59.4%	452.5

Variable Definitions

As stated above, the original PIA dataset includes a lot of detailed information for each firm. In this analysis, I focus on employment, investment, output and sector. The employment measure is given by the average of employees in the reference year of the survey. Investment is defined as the sum of the net acquisition of assets and the changes in stocks. Output is given by the industrial transformation value, which is the gross value of industrial production net of production costs. All nominal values are deflated to 2015 values using IPCA[5] data also released by IBGE.

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[5] Índice Nacional de Preços ao Consumidor Amplo – Broad National Consumer Price Index, in a direct translation from Portuguese.

Finally, each firm's sectoral identification is provided by the 3-digit CNAE[6] Code that is assigned yearly to each firm present in the dataset. To correct for possible measurement errors in the survey, e.g. a firm that changes from one sector to another in a given year and then moves back to its original sector or even to a different one, I impose that a firm's sector is the one where it spent most time in the sample, i.e. for each firm, its CNAE Code in the final sample is the one with the highest number of appearances between 1996 and 2015.

Building Firm-Level Capital Stock Series

In this section, I detail the steps needed to build a capital stock series $\{K_{ijt}\}_{t=0}^{T}$ for each firm i of sector j present in the balanced panel.

Following Doms and Dunne (1998), I use the perpetual inventory method to do so:

$$K_{ijt} = K_{ij(t-1)}\left(1-\delta_j\right) + I_{ijt}$$

where δ_j denote sector j's depreciation rate and I_{ijt} is firm i's current period investment. Then, note that we can rewrite it as

$$K_{ijt} = \left(1-\delta_j\right)^t K_{ij0} + \sum_{\tau=0}^{t}\left(1-\delta_j\right)^{\tau} I_{ij\tau}$$

As such, the method requires that a researcher interested in building the capital stock for any firm knows, besides the whole history of investment decisions, its initial capital stock K_{ij0} and the sectoral depreciation of its sector. Unfortunately, the Brazilian statistical office does not provide any of these informations, so said researcher has to compute these variables, which presents two simultaneous problems:

1. For each sector, what is the correct value of δ_j?

2. What is a plausible guess for K_{ij0}?

The simultaneity issue arises from the fact that, usually, estimates for δ_j are obtained from the estimation of the parameters of each sector's production function – which depends on K_{ij0} – and that most methods/guesses for initial capital stocks for firms (or countries) also depend on the depreciation rate associated with that capital stock.

Given all of this, I proceed in an iterative manner[7]. First, I guess K_{ij0} following King and Levine (1994) and imposing $\delta_j = 0.05$. Using this guess, I construct provisional capital

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[6] CNAE is an acronym for Classificação Nacional de Atividades Econômicas - National Economic Activity Classification, in a direct translation from Portuguese -, the Brazilian equivalent of NAICS used by the United States Census Bureau.

[7] Each one of these steps is carefully described in the appendix.

stock series for all firms and perform the constrained estimation of each sector's production function, which allows me to obtain estimates for δ_j and, evidently, the final guess for K_{ij0}. Finally, to reduce dependence on these procedures, I restrict the analysis to the period between 2000 and 2015.

3 Firm-Level Capital Accumulation and Investment Patterns

In possession of each firm's capital stock series, we can proceed to analyzing their behavior across time. In particular, I focus on two statistics and their distributions. The first one is the Growth Rate of Capital (GK), such as in Doms and Dunne (1998) and Davis and Haltinwanger (1992), which is determined for each firm i of sector j at time t by

$$GK_{ijt} = \frac{I_{ijt} - \delta_j K_{ij(t-1)}}{0.5 \cdot \left(K_{ij(t-1)} + K_{ijt}\right)}$$

One should note that $GK_{ijt} \in [-2, 2]$, where $GK_{ijt} = -2$ denotes the case where a firm exits the economy and, analogously, $GK_{ijt} = 2$ denotes the case where a firm is created[8]. This statistic allows us to understand how firms grow their capital stocks, while adjusting for firms sizes and giving a unique platform to think about births, deaths and continuing firms. In the case of the current analysis, I focus solely on continuing firms for the sake of simplicity but it is easily extendable if I relax some assumptions made during the data cleaning procedures. One other advantage of this measurement is that, as Davis and Haltinwanger highlight (1992), it is monotonically related to a standard growth rate and they are approximately equal for small growth rates.

The second measure of interest is simply the investment rate, which is defined as

$$i_{ijt} = \frac{I_{ijt}}{K_{ijt}}$$

which has the advantage of being stationary in standard models with balanced growth paths, which makes it a fundamental object to consider when mapping such models into and onto data, allowing thus for sensible quantitative analysis and within-model policy experiments.

Fact 1 – GK distribution is more dispersed, with heavier tails

Figure 1 presents the distribution of GK_{ijt}. It shows that the majority of firms, around 70%, increases its capital stock by less than 10%, while more than 20% of firms increase

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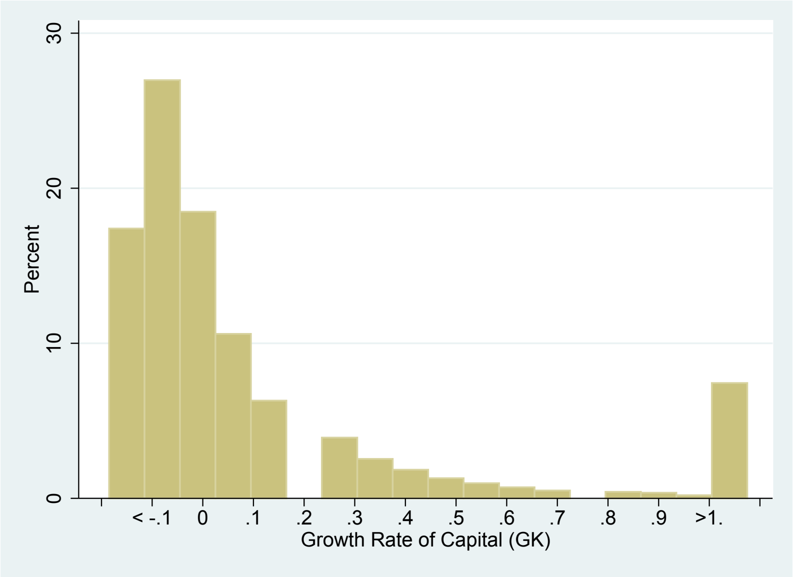
[8] Here, it is embedded the notion that a firm, to exist, must hold some of its own capital.

their capital stock by more than 20%. These numbers resemble those of the American distribution, but the percentages of firms at the left tail and at the right tail of distribution are strikingly higher in Brazil. On the left hand side of the distribution, more than 30% of firms exhibit negative capital growth rates against slightly more than 10% in the U.S. On the right hand side of the distribution, close to 10% of firms exhibit growth rates above 100% in Brazil while this number is smaller than 5% in the U.S.

Figure 1: Capital Growth Rate Distribution



This suggests that a small number of capital expansion episodes account for a relatively large share of investment, even larger than in the U.S, in the cross section[9]. However, this does not reveal much about lumpy behavior over time, which is to what we turn now.

Fact 2 – GK expansions and destructions are more intense

To think about within-firm capital accumulation patterns, I construct a rank as Doms and Dunne (1998). For each firm in the final balanced dataset, I rank, in descending order, the annual capital growth rates from highest to lowest considering each year between 2001 and 2015. Also as in the aforementioned article, I denote the growth rate in rank 1 by MaxGK. In Figure 2, I present the means and medians of these ranked growth rates. While 9 of the 15 ranks possess means or medians between -10% and +10%, MaxGK is significantly higher for Brazilian Firms, with means exceeding 50% and medians exceeding 40% against 46% and 36% in the U.S.

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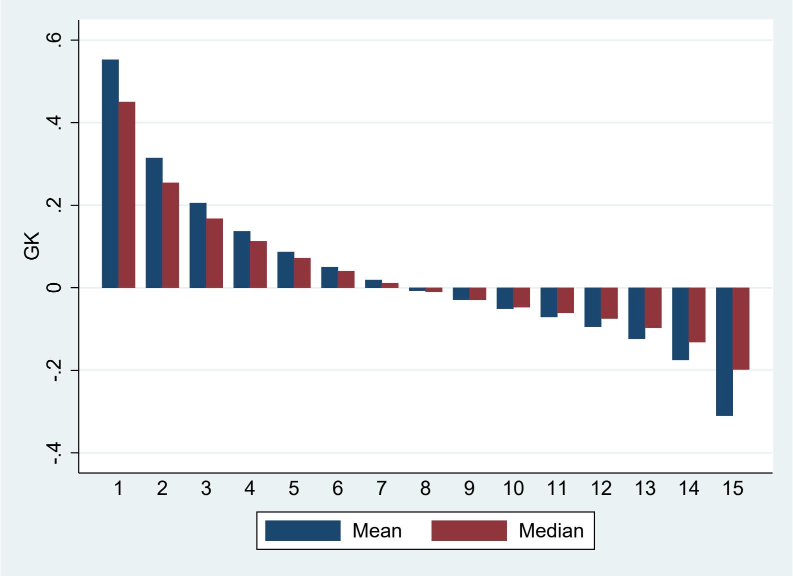
[9] I go back to this point in Fact 4.

Figure 2: Capital Growth Rates (GK) by rank, means and medians



Interestingly, as in the U.S., the analysis demonstrates meaningful variation across Brazilian firms in terms of capital growth rates. While the analysis also indicates that some firms experience sizable changes in their capital stock, such variation does not allow us to discard that some firms might experience smoother changes in their capital stocks. A possible explanation of the differences in such changes is that for some industries investment is inherently lumpy due of the nature of the capital goods which could arise due to the indivisibility of large machines, while for other industries it may be easier to adjust capital more smoothly. To examine this possibility, I model MaxGK for a firm as a function of size, number of local units, controlling for industry and other effects.

I estimate a regression model using all firms in the balanced panel. My firm-level measure of capital lumpiness is the maximum single year capital growth rate MaxGK. The regression also includes controls for firm size, which is modeled using a set of dummy variables representing plant-size quartiles. The quartiles go from smallest to largest, with the quartile representing the largest firms omitted. Finally, the regression is run with three-digit industry dummy variables, controlling for sectoral idiosyncrasies in the . I also include a dummy variable indicating whether the firm made the choice to be part of the special tax regime designed for micro and small firms. Unfortunately, PIA does not provide any information regarding firm age, so I do not include any proxies on this dimension. The results are exhibited in Table 2.

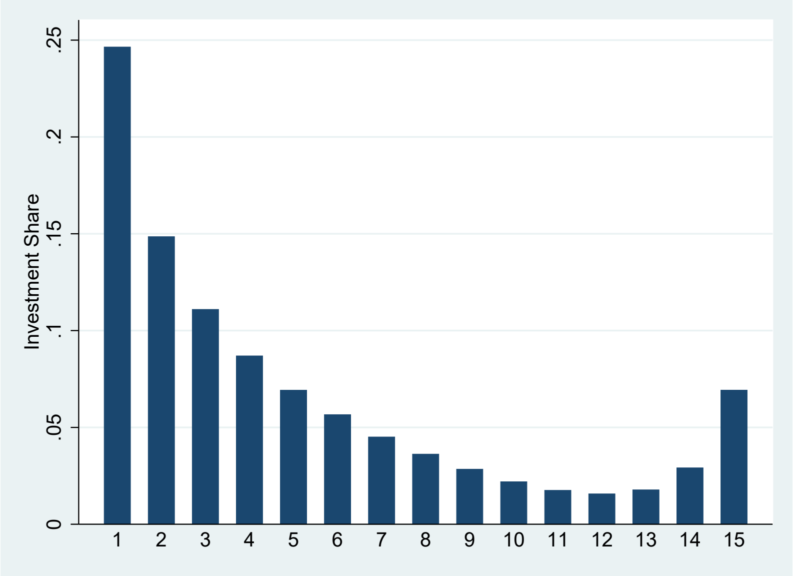
Table 2: Maximum Capital Growth Rate Regression: MaxGK is the DependentVariable

Variables	
Firm Size Quartile (Smallest to Largest)	
1st Quartile	0.152***
	(0.0195)
2nd Quartile	0.0979***
	(0.0186)
3rd Quartile	0.107***
	(0.0183)
Number of Local Units	-0.00100
	(0.000819)
Simples Dummy	0.0148
	(0.0252)
Observations	5,191
R^2	0.689
4th Quartile	No
106 3-Digit Industry Controls	Yes
15 Year Dummies in which MAXGK occurs	Yes

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

As in the U.S., smaller firms tend to have larger spikes, even after controlling for industry and other firm characteristics. Accordingly, as larger firms tend to have a larger number of local units, this fact is reinforced by the slightly negative coefficient relative to the number of local units. On another hand, in Brazil, these firm characteristics seem explain a lot more of the size of MaxGK. On the flip-side of these single large expansions of their capital stocks, it is also striking that, on average, firms tend to experience way larger disinvestment periods when compared to the American establishments, where the mean and the median never go lower than -10%. This point is ratified by Figure 3, where I display the mean investment share according to this rank.

Figure 3: Mean Investment Shares by Capital Growth Rate rank.



To obtain the investment share, I aggregate total investment by each firm and, then, calculate a simple ratio of investment in a specific period over total investment. Then, for each rank of the growth rate of capital, I take the average of the investment share across firms, thus obtaining the mean investment share by each of the 15 ranks. For an example, the bar that corresponds to rank 1 shows that the average firm experiences almost 20% of its total investment in the year in which they experience the largest growth rate of their stock of capital. Given a substantial number of firms with decreases in the stock of capital over the period of analysis, we observe a reversal of this ordering, i.e. a lot of firms have most of their (dis)investment in the period where they exhibited the smallest growth rate of their capital stocks.

Fact 3 – Investment Rate Distribution has more dispersion, inaction and spikes

In this subsection, I characterize the investment rate distribution in Brazil, presenting also a comparison the findings for the American firms found in Zwick and Mahon (2017) and Winberry (2021). To do so, I compute a few statistical measures that try to capture not only the usual moments of interest - average and standard deviation - for any distribution, but also a few moments related to lumpiness of investment.

There are three statistical measures of investment lumpiness: inaction rate, spike rate, positive investment rate. The inaction rate is defined as the fraction of observations with investment rate less than 1%. The spike rate is the fraction of observations with investment rate greater than 20%. Finally, positive investment rate is the fraction of observations with investment rate between 1% and 20%. Table 3 exhibits these measures, pooled over firms and time, both for Brazil and United States.

Table 3: Micro Lumpy Investment Statistics

	Brazil (2000-2015)	United States (1998-2010)
Statistic	Value	Value
Inaction Rate (%)	29.8%	23.7%
Spike Rate (%)	23.6%	14.4%
Positive Investment Rate (%)	46.6%	61.9%
Average Investment Rate (%)	5.8%	10.4%
S.D. of Investment Rates	3.2	0.2

Statistics drawn from distribution of investment rates pooled over firms and time. Inaction rate is fraction of observations with investment rate less than 1%. Spike rate is fraction of observations with investment rate greater than 20%. Positive investment is fraction of observations with investment rate between 1% and 20%

They indicate that there are important differences in investment dynamics in Brasil vis-à-vis the U.S. First, investment rates are lower and the dispersion is significantly higher in Brazil than in the U.S, a result that might have different interpretations. On one hand, it could be the case, as argued by Hsieh and Klenow (2009) in a completely different setting, that inefficient firms tend to survive more in Brazil if compared to the United States and, as inefficient firms are unlikely to grow much over time, they would, on average, invest less, accumulating less capital and also diminishing the average investment rate. On the other hand, it is also possible that other constraints, of either financial or physical adjustment nature, are tighter in Brazil, what could also be related to the other important difference of the investment rate distribution in Brazil and in the U.S.

The second important difference between the Brazilian and the American investment rate is that Brazilian firms experience, on average, more episodes of investment spikes and they also tend to, on average, have more periods without any investment. This finding is in line with the higher standard deviation of investment rates and with the previous facts, where I showed that the growth rate of capital distribution has fatter tails and exhibits higher dispersion than in the U.S.

More details about the evolution of this distribution over this period can be found in the appendix[10]. The most striking fact is how it is subject to changes across the entire period, in particular in response to large aggregate economic shocks such as in the aftermath of the Great Recession and of the most recent economic crisis in Brazil (2014–2015), with an increase in inaction, decrease in positive investment and spike investment and, as a consequence, of average investment rates.

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[10] See subsection A.2.

Fact 4 – Factors of Production are very concentrated

Facts 1–3 suggest that a small number of investment episodes are responsible for most of aggregate investment. To reinforce this point, I constructed the table below, which presents a snapshot of the shares of investment, employment, output and capital stock that are held by the biggest firms in each category in 2007 and 2015. It suggests indicates that not only capital and investment are concentrated on a small number of firms, but also that economic activity is concentrated on a small number of firms.

Table 3: Share of Investment, Employment, Output and Capital Accounted for by the Top Firms in Each Category (5191 Firms)

	2007			
	Investment	Employment	Output	Capital Stock
Top 10 Firms	59.7%	10.1%	40.8%	50.7%
Top 50 Firms	73.7%	23.0%	56.6%	64.6%
Top 100 Firms	80.6%	31.1%	64.5%	71.5%
Top 200 Firms	87.4%	41.3%	72.3%	78.3%
Top 500 Firms	94.9%	58.5%	82.4%	87.3%
Top 1000 Firms	99.3%	72.1%	89.7%	93.1%
Top 1500 Firms	101.2%	80.1%	93.4%	95.9%
	2015			
	Investment	Employment	Output	Capital Stock
Top 10 Firms	65.3%	10.7%	39.4%	62.0%
Top 50 Firms	78.3%	25.5%	56.3%	72.3%
Top 100 Firms	85.0%	34.2%	64.7%	77.8%
Top 200 Firms	91.5%	44.9%	72.5%	83.7%
Top 500 Firms	99.0%	62.2%	83.0%	90.6%
Top 1000 Firms	103.0%	75.5%	90.5%	95.1%
Top 1500 Firms	104.5%	83.0%	94.2%	97.1%

4 Model

In the previous section, I showed how micro-level lumpiness of investment is even more pronounced amongst Brazilian firms than amongst American firms, with other relevant differences. Since the seminal contribution by Thomas (2002), economists have investigated whether this matters to understand the behavior of macroeconomic aggregates, which requires a proper model that can account for general equilibrium effects.

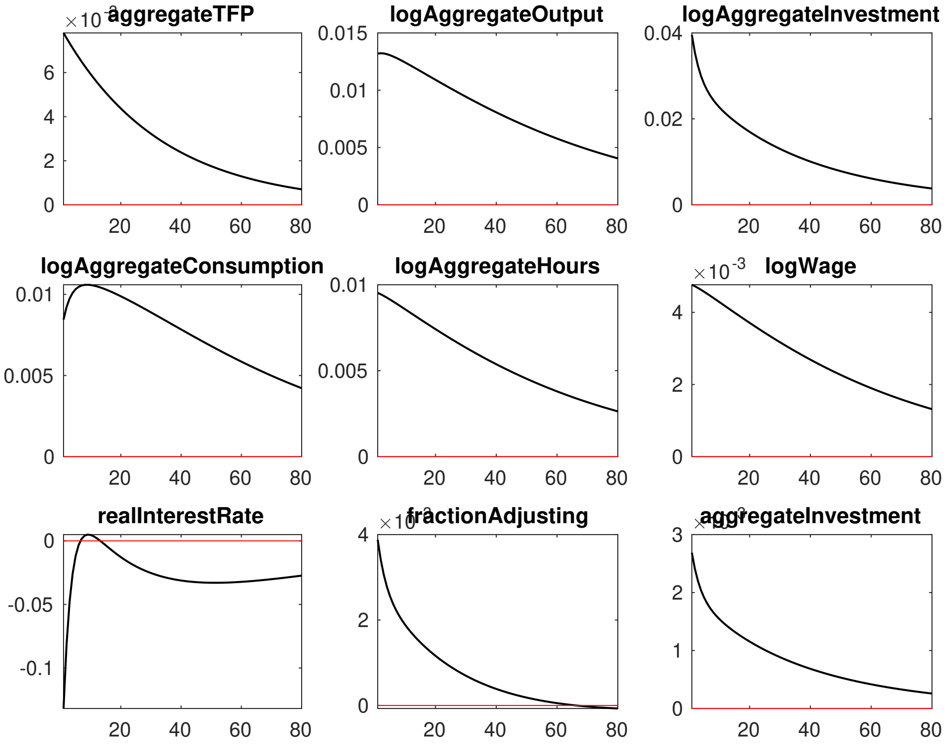
To be precise, the behavior of aggregate investment in response to aggregate shocks depends on the interaction of supply, which are disciplined by household preferences over consumption, and demand forces, which are disciplined by the structure of capital adjustment costs. Standard RBC models with firm heterogeneity, such as Khan and Thomas (2008), suggest that general equilibrium forces revert the tension generated by infrequent micro-adjustment of capital stocks. However, as pointed out by Winberry (2021), these models imply pro-cyclical dynamics for real interest rates, a fact which is at odds with the data.

In what follows, I make the same assumptions about the environment as Winberry (2021). This is plausible since, even though a lot of economists might claim that Brazil should be modeled as a small open economy, the Brazilian economy is relatively closed, one of the least open economies in the entire G20, and most Brazilian firms do not directly rely on international credit. Additionally, real interest rates in Brazil exhibit the same countercyclical behavior, as pointed out by Kanczuk (2004), Niemeyer and Perri (2005), Segura-Ubiergo (2012) and Souza-Sobrinho (2011).

Environment

In this section, I outline the model, which an extension of the standard RBC model to account for firm heterogeneity, such as in Winberry (2021) and Khan and Thomas (2008). Time is discrete, indexed by by $t = 0,\ 1,\ 2, \dots$, and the horizon is infinite. There is a continuum of heterogeneous production units, indexed by j and distributed uniformly over $[0, 1]$. Each firm is subject to an aggregate productivity shock which is common to all firms, driving business cycle fluctuations, and it is also subject to an idiosyncratic shock which generates heterogeneity in the investment pattern across firms and across time. On the household side, there is a single representative agent with preferences that exhibit habit formation over consumption. With these assumptions, we can match the behavior of interest rates as in the data, which in turn means that aggregate investment has a strong response to a positive aggregate shock (TFP shock) as we can note the last row in Figure 4. Finally, the model is closed with a government that taxes firms' profits and transfers the proceed lump sum to the household.

Figure 4: Model Responses to a Positive TFP Shock



Firms

All firms have the same production technology that combines labor and capital in order to produce output. Specifically, we assume that a firm j at time t produces y_{jt} according to:

$$y_{jt} = e^{z_t} e^{\varepsilon_{jt}} k_{jt}^{\theta} n_{jt}^{\nu},\ \theta + \nu < 1$$

where z_t is the aggregate productivity shock, ε_{jt} is the idiosyncratic shock, k_{jt} is capital, n_{jt} is labor, and $\theta$and ν are parameters. We assume that z_t and ε_{jt} follow two different $AR\left(1\right)$ processes and that ε_{jt} is independent across firms.

At each period, a firm j observes these two shocks, uses its pre-existing capital stock, hires labor from a competitive market and, then, produces output. After production, the firm makes its investment decision regarding the next period and such investment is subject to two adjustment costs, a fixed cost, given by ξ_{jt} which is uniformly distributed over $\left[0, \overline{\xi}\right]$ and is measured in units of labor, and a variable cost, given by $-\frac{\phi}{2}\left(\frac{i_{jt}}{k_{jt}}\right)^2 k_{jt}$ in units of output. There is also some degree of flexibility in the adjustment cost function as I allow the firm not to pay the fixed cost if it invests $i_{jt} \in \left[-ak_{jt}, ak_{jt}\right]$. If it invests some value outside of this interval, then the firms has to pay the fixed cost. After production and investment, the firm pays a linear tax τ on its revenue y_{jt} net of labor costs, $w_t n_{jt}$, and capital depreciation costs, which is deducted in geometric schedule and takes into account a constant fraction, $\hat{\delta}$, of its pre-existing stock of depreciation allowances d_{jt} and of its new investment i_{jt}.

Household

As I mentioned above, there is a representative household with consumption preferences that exhibit habit formation over consumption. This representative household has expected utility function given by

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t \log \left(C_t - H_t + \chi \frac{N_t^{1+\eta}}{1+\eta} \right)$$

where C_t is consumption, H_t is habit stock and N_t is labor supplied to the market.

The habit stock H_t is defined to capture the idea that the utility of current consumption is judged relative to past consumption. Let $S_t = \frac{C_t - H_t}{C_t}$ be the surplus consumption ratio and, then, specify the law of motion

$$\log S_{t+1} = (1-\rho_s) \log \bar{S} + \rho_s \log S_t + \lambda \log \frac{C_{t+1}}{C_t} \tag{1}$$

implying that current habit is approximately a geometric average of past consumption.

I also assume that total time endowment per time is 1, so that $N_t \in [0,1]$. The household owns all firms in the economy and markets are complete.

Government

The government collects the corporate profits from the firms and transfers the proceeds lump sum to the household. Supposing that its budget is always balanced, we have that, in period t, the transfer is given by

$$T_t = \tau \left(Y_t - w_t N_t - \hat{\delta} \left(D_t + I_t \right) \right), \tag{2}$$

where Y_t is aggregate output, N_t is the aggregate labor input, D_t is the aggregate stock of depreciation allowances and I_t is aggregate investment.

Equilibrium

In order to define the equilibrium of this economy, we must first discuss each sector's problem and the aggregate state vector. The aggregate state vector s_t is fundamental in the equilibrium as it defines the prices that firms take as given, the habit stock, the government transfer and firms profits that are key to solving the household's problem. At any given period t, the aggregate state vector is given by $\mathrm{s}_t = (z_t,\ S_{t-1},\ C_{t-1},\ \mu_t)$[11], where μ_t is the distribution of firms over their individual state vector $(\varepsilon_{it},\ k_{it},\ \xi_{it})$.

Firm's Problem. I characterize the optimization problem of a firm recursively. Evidently, the firm's individual state variables are given by ε_{jt}, k_{jt}, d_{jt} and ξ_{jt}, which we defined above. As I also discussed above, firms also take prices as given, as they are determined by the aggregate state vector s_t. Thus, the firm's value function, $v\left(\varepsilon,\ k,\ d,\ \xi;\ \mathrm{s}\right)$, solves the Bellman equation

$$\begin{aligned} v\left(\varepsilon,\ k,\ d,\ \xi;\ \mathrm{s}\right) = r\hat{\delta}d + \max_{n} &\left\{(1-\tau)\left(e^{z}e^{\varepsilon}k^{\theta}n^{\nu} - w\left(\mathrm{s}\right)n\right)\right\} \\ &+ \max\left\{v^{a}\left(\varepsilon,\ k,\ d,\ \xi;\ \mathrm{s}\right) - \xi w\left(\mathrm{s}\right),\ v^{n}\left(\varepsilon,\ k,\ d,\ \xi;\ \mathrm{s}\right)\right\}. \end{aligned} \tag{3}$$

The first max operator represents the optimal choice of labor and the second max operator represents the optimal choice of investment. These choices can are independent as the choice of labor is a purely static decision.

The second max operator represents the fact that the firm must choose between adjusting or not adjusting its capital stock. If it chooses to adjust, then it must pay its fixed cost $-\xi w\left(\mathrm{s}\right)$ and achieves the choice-specific value function $v^{a}\left(\varepsilon,\ k,\ d,\ \xi;\ \mathrm{s}\right)$, which is defined by the following Bellman equation:

$$\begin{aligned} v^{a}\left(\varepsilon,\ k,\ d,\ \xi;\ \mathrm{s}\right) = \max_{i\in\mathbb{R}} - &\left(1 - r\hat{\delta}\right)i - \frac{\phi}{2}\left(\frac{i}{k}\right)^{2}k \\ &+ \mathbb{E}\left[\Lambda\left(z';\ \mathrm{s}\right)v\left(\varepsilon',\ k',\ d',\ \xi';\ \mathrm{s}'\right)|\varepsilon,\ k,\ d\right] \\ &\text{s.t. } k' = (1-\delta)k + i \\ &\quad d' = \left(1-\hat{\delta}\right)(d+i) \end{aligned} \tag{4}$$

where $\Lambda\left(z';\ \mathrm{s}\right)$ is the stochastic discount factor. On the other hand, if the firm chooses not to pay its fixed cost, it achieves the choice specific value function $v^{n}\left(\varepsilon,\ k,\ d,\ \xi;\ \mathrm{s}\right)$, which

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[11] When defining the equilibrium, we will drop the time subscript.

is defined by the following Bellman equation:

$$\begin{aligned} v^{n}\left(\varepsilon,\ k,\ d,\ \xi;\ \mathbf{s}\right) = \max_{i\in[-ak,ak]} & -\left(1-r\hat{\delta}\right)i-\frac{\phi}{2}\left(\frac{i}{k}\right)^{2}k \\ & +\mathbb{E}\left[\Lambda\left(z';\ \mathbf{s}\right)v\left(\varepsilon',\ k',\ d',\ \xi';\ \mathbf{s}'\right)|\varepsilon,\ k,\ d\right] \\ & \text{s.t. } k'=\left(1-\delta\right)k+i \\ & d'=\left(1-\hat{\delta}\right)\left(d+i\right) \end{aligned} \tag{5}$$

The only difference between equations (4) and (5) is that, when the firm does not pay its fixed cost, investment is constrained to be in the set $[-ak, ak]$. Thus, the firm will choose to pay its fixed cost if and only if $v^{a}\left(\varepsilon,\ k,\ d,\ \xi;\ \mathbf{s}\right)-\xi w\left(\mathbf{s}\right)\geq v^{n}\left(\varepsilon,\ k,\ d,\ \xi;\ \mathbf{s}\right)$. For each state vector, one can show that there is a unique fixed cost threshold that makes the firm indifferent between these two options and, as such, the randomness of the fixed cost will generate infrequent decisions of adjustment. This is exactly what generates the lumpy investment patterns that we observe in the data.

Mathematically, the fixed cost threshold is given by

$$\hat{\xi}\left(\varepsilon,\ k,\ d,\ \mathbf{s}\right)=\frac{v^{a}\left(\varepsilon,\ k,\ d,\ \xi;\ \mathbf{s}\right)-v^{n}\left(\varepsilon,\ k,\ d,\ \xi;\ \mathbf{s}\right)}{w\left(\mathbf{s}\right)} \tag{6}$$

Household's Problem. As the investment decision is done by the firms, there are no dynamic links in the household's choices and we can model its decision problem statically. Thus, the household problem, at a given aggregate state s, is given by:

$$\begin{aligned} \max_{C,\,N}\ \log\left(C-H\left(\mathbf{s}\right)-\chi\frac{N^{1+\eta}}{1+\eta}\right). \\ \text{s.t. } C\leq w\left(\mathbf{s}\right)N+\Pi\left(\mathbf{s}\right)+T\left(\mathbf{s}\right) \end{aligned} \tag{7}$$

The budget constraint satisfies that total expenditure, given by consumption, C, can not surpass total income, given by labor income, $w\left(\mathrm{s}\right)N$, profits from owning the firms, $\Pi\left(\mathrm{s}\right)$, and the lump sum transfer from the government, $T\left(\mathrm{s}\right)$. As Winberry (2021) highlights, even though the household does not have investment decisions per se, the fact that markets are complete implies that stochastic discount factor used by firms to price investment is equal to the household's intertemporal marginal rate of substitution state by state:

$$\Lambda\left(z';\ \mathbf{s}\right)=\frac{C\left(\mathbf{s}\right)\times S\left(\mathbf{s}\right)-\chi\frac{N(\mathbf{s})^{1+\eta}}{1+\eta}}{C\left(\mathbf{s}'\right)\times S\left(\mathbf{s}'\right)-\chi\frac{N(\mathbf{s}')^{1+\eta}}{1+\eta}} \tag{8}$$

As argued before, these preferences combined with the firms' capital adjustment costs allow the model to generate a countercyclical interest rate such as we observe in the data.

Equilibrium. We can finally proceed to the equilibrium.

Definition. A recursive competitive equilibrium for this economy is a a list of functions $v\left(\varepsilon,\ k,\ d,\ \xi;\ \mathrm{s}\right)$, $n\left(\varepsilon,\ k,\ d,\ \xi;\ \mathrm{s}\right)$, $i^{a}\left(\varepsilon,\ k,\ d;\ \mathrm{s}\right)$, $i^{n}\left(\varepsilon,\ k,\ d;\ \mathrm{s}\right)$, $\hat{\xi}\left(\varepsilon,\ k,\ d;\ \mathrm{s}\right)$, $C\left(\mathrm{s}\right)$, $N\left(\mathrm{s}\right)$, $T\left(\mathrm{s}\right)$, $w\left(\mathrm{s}\right)$, $\Pi\left(\mathrm{s}\right)$, $\Lambda\left(z';\ \mathrm{s}\right)$, $S'_{-1}\left(\mathrm{s}\right)$, $C'_{-1}\left(\mathrm{s}\right)$, and $\mu'\left(\mathrm{s}\right)$ -such that

1. (Household's Problem) Taking $T\left(\mathrm{s}\right)$, $w\left(\mathrm{s}\right)$, $\Pi\left(\mathrm{s}\right)$ as given, $C\left(\mathrm{s}\right)$and $N\left(\mathrm{s}\right)$solve the utility maximization problem given by (7).
2. (Firm's Problem) Taking $w\left(\mathrm{s}\right)$, $\Lambda\left(z';\ \mathrm{s}\right)$, $C'_{-1}\left(\mathrm{s}\right)$, and $\mu'\left(\mathrm{s}\right)$ as given, $v\left(\varepsilon,\ k,\ d,\ \xi;\ \mathrm{s}\right)$, $n\left(\varepsilon,\ k,\ d,\ \xi;\ \mathrm{s}\right)$, $i^{a}\left(\varepsilon,\ k,\ d;\ \mathrm{s}\right)$, $i^{n}\left(\varepsilon,\ k,\ d;\ \mathrm{s}\right)$and $\hat{\xi}\left(\varepsilon,\ k,\ d;\ \mathrm{s}\right)$solve the firm's maximization problem given by $(3)-(6)$.
3. (Government) For all s, $T\left(\mathrm{s}\right)$ is given by (2).
4. (Consistency)
   (a) The profit function is given by

$$\begin{aligned}\Pi\left(\mathrm{s}\right)=\int[&\left(1-\tau\right)\left(e^{z}e^{\varepsilon}k^{\theta}\left(n\left(\varepsilon,\ k,\ d,\ \xi;\ \mathrm{s}\right)\right)^{\nu}-w\left(\mathrm{s}\right)n\left(\varepsilon,\ k,\ d,\ \xi;\ \mathrm{s}\right)\right)+r\hat{\delta}d\\&-\left(1-\tau\hat{\delta}\right)i\left(\varepsilon,\ k,\ d;\ \mathrm{s}\right)-\frac{\phi}{2}\left(\frac{i\left(\varepsilon,\ k,\ d;\ \mathrm{s}\right)}{k}\right)^{2}k-\xi w\left(\mathrm{s}\right)1\left\{\frac{i\left(\varepsilon,\ k,\ d;\ \mathrm{s}\right)}{k}\notin[-a,a]\right\}]\\&\mu\left(d\varepsilon,\ dk,\ dd,\ d\xi\right),\end{aligned}$$

$$\text{where } i\left(\varepsilon,\ k,\ d;\ \mathrm{s}\right)=\begin{cases}i^{a}\left(\varepsilon,\ k,\ d;\ \mathrm{s}\right) & ,\ if\ \xi\leq\hat{\xi}\left(\varepsilon,\ k,\ d;\ \mathrm{s}\right)\\ i^{n}\left(\varepsilon,\ k,\ d;\ \mathrm{s}\right) & ,\ otherwise.\end{cases}$$

   (b) $\Lambda\left(z';\ \mathrm{s}\right)$ is given by (8).
   (c) $S'_{-1}\left(\mathrm{s}\right)$ follows (1).
   (d) $C'_{-1}\left(\mathrm{s}\right)=C\left(\mathrm{s}\right)$.
   (e) For all measurable sets $\Delta_{\varepsilon}\times\Delta_{k}\times\Delta_{d}\times\Delta_{\xi}$,

$$\begin{aligned}\mu'\left(\Delta_{\varepsilon}\times\Delta_{k}\times\Delta_{d}\times\Delta_{\xi}\right)=\int&p\left(\varepsilon'\in\Delta_{\varepsilon}|\varepsilon\right)d\varepsilon'\times\mathbb{I}\left\{i\left(\varepsilon,\ k,\ d;\ \mathrm{s}\right)+\left(1-\delta\right)k\in\Delta_{k}\right\}\\&\times1\left\{i\left(\varepsilon,\ k,\ d;\ \mathrm{s}\right)+\left(1-\delta\right)k\in\Delta_{k}\right\}\\&\times1\left\{\left(1-\hat{\delta}\right)i\left(\varepsilon,\ k,\ d;\ \mathrm{s}\right)+d\in\Delta_{d}\right\}\\&\times G\left(\Delta_{\xi}\right)\mu\left(d\varepsilon,\ dk,\ dd,\ d\xi\right)\end{aligned}$$

where $G(\xi)$ is the CDF of ξ.

5. (Market Clearing) For all s, $N(\mathbf{s}) = \int n(\varepsilon,\ k,\ d,\ \xi;\ \mathbf{s})\, \mu(d\varepsilon,\ dk,\ dd,\ d\xi)$.

The mapping in fifth condition of the consistency condition of the equilibrium defines the measure of firms in the set $\Delta_\varepsilon \times \Delta_k \times \Delta_d \times \Delta_\xi$ next period in terms of the distribution of firms and individual decisions in the current period. Intuitively, this mapping counts up the mass of individual states in the current period which leads into the set $\Delta_\varepsilon \times \Delta_k \times \Delta_d \times \Delta_\xi$ next period.

Solution Method

The presence of both autocorrelated idiosyncratic shocks to firms' individual productivity and an autocorrelated aggregate TFP shock imposes that we need to keep track of the entire distribution of firms over the individual states $\mu(d\varepsilon,\ dk,\ dd,\ d\xi)$, which like in Krusell & Smith (1998) – KS – is a computationally challenging procedure. To deal with this, I rely on Winberry (2018), which posits that, instead of approximating the distribution with moments, one can approximate it with a flexible but finite-dimensional parametric family, whose the parameters are actually endogenous aggregate state variables of the approximated model.

This approach makes the problem substantially faster to solve, given the use of perturbations to solve this approximation problem, but it also casts doubt whether our solution is indeed globally accurate, which would be the case as in KS. However, a linear law of motion based solely on aggregate capital as in KS fails to determine aggregate dynamics, which should be understood as this economy not having a near-aggregation result.

Calibration

In order to solve the model numerically, we have to attribute values to 16 parameters, which are displayed in table 4.

Table 4: Parameters

Fixed Parameters			**Source**
β	Discount factor	0.99	Winberry (2021)
η	Inverse Frisch elasticity	0.50	Winberry (2021)
θ	Labor share	0.55	Cavalcanti et al (2021)
ν	Capital share	0.4	Cavalcanti et al (2021)
δ	Capital depreciation	0.05	Data
ρ_z	TFP shock persistence	0.97	Winberry (2021)
σ_z	TFP shock standard deviation	0.0078	Winberry (2021)
τ	Tax rate	0.29	Ulyssea (2018)
$\hat{\delta}$	Tax depreciation	0.04	Brazilian Tax Code
"Fitted" Parameters			**Source**
Micro Heterogeneity			
$\bar{\xi}$	Upper bound on fixed costs	0.80	–
a	Size of no fixed cost region	0.00	(Fixed)
ϕ	Quadratic adjustment	3.50	–
ρ_ε	Idiosyncratic shock persistence	0.9	(Fixed)
σ_ε	Idiosyncratic shock standard deviation	0.075	–
Habit Formation			
$\bar{S}$	Average surplus consumption	0.65	Winberry (2016)
ρ_S	Autocorrelation of surplus consumption	0.95	Winberry (2016)

Validation

The model overestimates average investment rates, understestimates standard deviation rates and it does not generate enough spike periods.

Table 5: Investment Empirical Targets

Target	Data	Model
Average Investment Rate (%)	5.75%	~ 8%
S.D. of Investment Rates	2.16	~ 0.6
Spike Rate (%)	23.6%	~ 15%
"Positive Investment" Rate (%)	76.4%	~ 85 %

5 Final Remarks

In this article, I take advantage of a comprehensive dataset originated from a Brazilian industrial survey to fill a void in the investment related literature with regard to the understanding of firm-level investment decisions in developing countries. Even though the broad picture is similar to the one regarding the U.S economy, i.e. evidence of micro-level lumpiness of investment, I document that there are some important differences in this panel of Brazilian firms.

First, I document that the distribution of the growth rate of capital is more dispersed, with “fatter” tails. Second, I show that episodes of capital expansion and destruction are more intense in Brail. Third, I compute statistical measures related to the investment rate distribution. These measures show investment at the firm-level seems to be even lumpier in Brazil, with firms investing less on average, while experiencing more episodes of investment spikes and periods of inaction. Fourth, I show that factors of production are highly concentrated, even more than in US.

Then, I make sense of some of these facts by solving and calibrating a RBC model with heterogeneous firms that face both convex and non-convex adjustment costs to adjusting their capital stocks. The model suggests that accounting for these micro-level irregularities is an important step to understand macroeconomic dynamics.

Further research should focus on improving the quantitative fit of the model to the data and also on running counterfactual analysis with this improved version of the model in order to understand better what are the possible limitations or benefits of stimulus policies in a developing market economy.

A Appendix

A.1 Building Firm-Level Capital Stock Series

First Guess of Initial Capital Stocks

To guess the initial capital stock of each firm, I follow the work of King and Levine (1994). The following steps are repeated after the estimation of the depreciation rates, but, as this is the only difference, I will only present them once.

These authors compute initial capital by using a steady-state method:

$$K_{ij2000} = \underline{\kappa}_{ij} Y_{ij2000}$$

where Y_{ij2000} is firm i's output in the initial period and $\underline{\kappa}_{ij}$ is the steady-state capital-output ratio, which is given by $\underline{\kappa}_{ij} = \underline{i}_{ij} / \left[\delta_j + \underline{\gamma}_{ij}\right]$, where $\underline{i}_{ij}$ is firm i's steady-state investment rate and $\underline{\gamma}_{ij}$ is firm i's steady-state growth rate.

For the first guess, I assume a single depreciation rate of $\delta = 0.05$ across all sectors, i.e. $\delta_j = \delta = 0.05,\ \forall\, j$. I also assume that the steady-state investment rate is the same for all firms and it is given by $\underline{i} = \underline{i}_{ij} = 10.4\%,\ \forall\, i, j$, the average investment rate of American firms in the sample of Zwick and Mahon (2017).

On the other hand, to compute the steady-state growth rate of firm i, I use a weighted average growth rate. To be more precise, I define the steady-state growth rate of firm i as

$$\underline{\gamma}_{ij} = \lambda \gamma_{ij} + (1 - \lambda)\, \gamma_w$$

where γ_i is firm i's growth rate over the period of 2000-2015, γ_w is the average growth rate in the panel of firms and λ is a parameter that measures how much weight each firm has on the steady-state. Following King and Levine (1994), I set $\lambda = 0.25$.

Estimating Depreciation Rates

After setting a first guess for the initial capital stock for each firm, I proceed with estimation of each sector's depreciation rate following the work of Doms (1996) with a few modifications. First of all, I assume a Cobb-Douglas production function[12]. Second, I constrain the estimates of δ_j to the interval $[0.03,\ 0.15]$ for each sector j. At last, I only estimate the depreciation rate for sectors where at least 30 firms appear, setting the depreciation rate at 0.05 for sectors with a number of firms that is smaller than this threshold.

As such, for each sector j that has at least 30 firms, I estimate using Nonlinear Least Squares the following equation:

$$\ln Y_{ijt} = \beta_0 + \beta_L \ln L_{ijt} + \beta_K \ln \left(\underbrace{(1-\delta_j)^t K_{ij0} + \sum_{\tau=0}^{t} (1-\delta_j)^{\tau} I_{ij\tau}}_{K_{ijt}} \right) + \gamma_t D_t + \varepsilon_{ijt}$$

where Y_{ijt} is output, L_{ijt} is labor, I_{ijt} is investment and ε_{jt} is an i.i.d error term, with the subscript i referring to firm i in sector j. The term $\gamma_t D_t$ is meant to capture technical change that might have happened in each year. In total, the final sample has 107 sectors identified through 3-digit industry codes (3-digit CNAE number).

In Table 6, I present the results of the regression for sector 25 in the next page. In figure 1, I show how the distribution of depreciation rates looks like across sectors.

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[12] Doms (1996) estimates a translog production function. Due to time constraints and computational difficulties, I decided to estimate a simpler production function.

Table 6: Efficiency Schedule: Estimating a Cobb-Douglas Production Function with Geometric Efficiency for Sector 25

Dependent Variable = $\ln$(output)			
β_0	8.492***	γ_{2006}	0.315***
	(0.116)		(0.103)
β_L	1.150***	γ_{2007}	0.386***
	(0.0181)		(0.103)
β_K	0.0506***	γ_{2008}	0.429***
	(0.00485)		(0.103)
δ_{25}	0.0490***	γ_{2009}	0.556***
	(0.00137)		(0.103)
γ_{2001}	0.220**	γ_{2010}	0.552***
	(0.103)		(0.103)
γ_{2002}	0.0548	γ_{2011}	0.524***
	(0.103)		(0.103)
γ_{2003}	0.175*	γ_{2012}	0.531***
	(0.103)		(0.103)
γ_{2004}	0.179*	γ_{2013}	0.546***
	(0.103)		(0.103)
γ_{2005}	0.266**	γ_{2014}	0.510***
	(0.103)		(0.103)
		γ_{2015}	0.518***
			(0.103)
R^2	0.588		
Observations	3985		

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

In figure 5, I show how the distribution of depreciation rates across sectors looks like. Given the constraints on the values of δ_j whenever the estimation is performed and also on the other restrictions imposed prior to the estimation, there is a concentration of estimates on the bounds of the estimation and also around the default value of $\delta_j = 0.05$. In any case, there seems to be substantial heterogeneity across sectors, which indicates that this is an important feature to be taken into account whenever constructing capital stock series for firms.

Figure 5: Distribution of Depreciation Rates

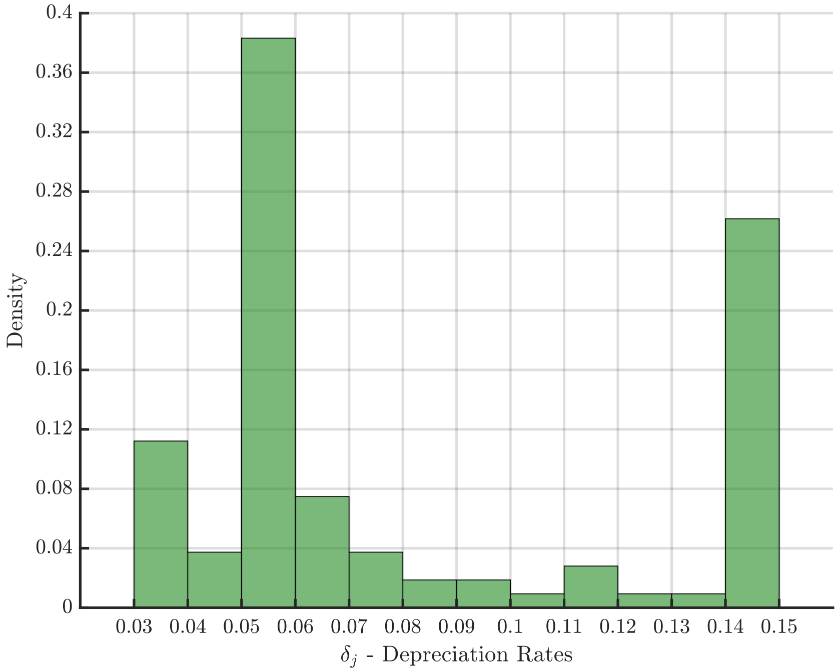
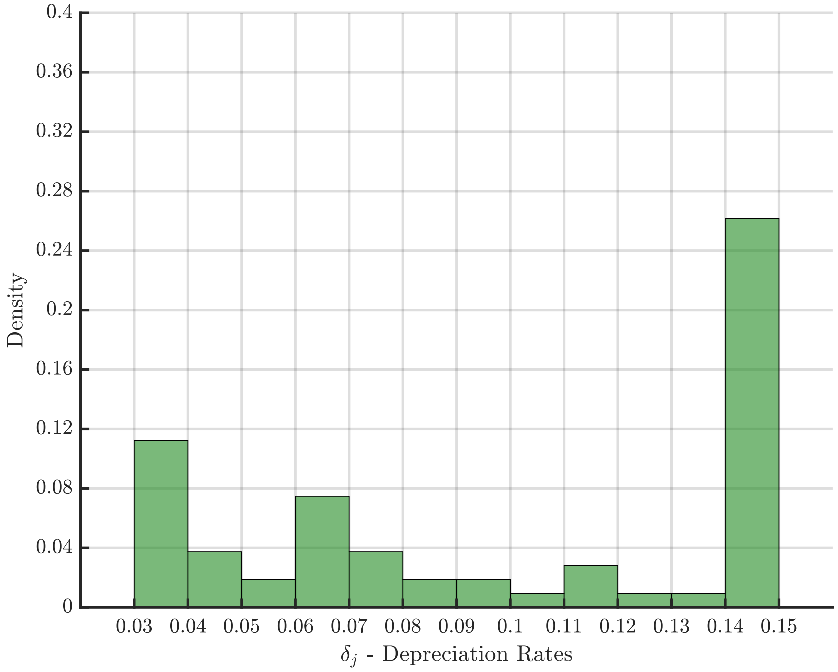


Figure 6 excludes the depreciation rates that were imputed, i.e. the ones for the sectors in which I have less than 30 firms.

Figure 6: Distribution of Depreciation Rates – Ex-Imputed δ_j's



Final Step: Capital Stock Series for each firm

At last, in possession of each sector's depreciation rate and recalculating the initial capital stock for each firm, I build each firm's capital stock using the perpetual inventory method. Computationally, I proceed in an iterate manner.

1. Using the new guess of initial capital stock for each firm and each firm's investment in the year 1996, I compute its capital stock in 1997.
2. In possession of this value and the investment in 1998, I compute the capital stock in 1998.
3. This procedure is repeated until I compute the capital stock in 2015.

To diminish results' dependence on initial conditions, I restrict the analysis to the period between 2000 and 2015.

A.2 Investment Rate Distribution – Details per Year

Table 6: Yearly Micro Lumpy Investment Statistics

	Value				
Statistic	2000	2001	2002	2003	2004
Inaction Rate (%)	24.0%	25.0%	27.3%	27.3%	27.1%
Spike Rate (%)	29.1%	28.0%	23.8%	24.5%	26.4%
Positive Investment Rate (%)	46.9%	47.0%	48.9%	48.1%	46.5%
Average Investment Rate (%)	11.9%	9.4%	9.7%	7.8%	7.0%
S.D. of Investment Rates	0.6	1.0	0.4	1.2	3.1

	Value				
Statistic	2005	2006	2007	2008	2009
Inaction Rate (%)	29.0%	29.9%	27.5%	26.6%	36.3%
Spike Rate (%)	24.3%	22.7%	25.8%	28.6%	19.7%
Positive Investment Rate (%)	46.7%	47.4%	46.7%	44.8%	44.0%
Average Investment Rate (%)	9.3%	8.1%	7.5%	11.6%	-5.7%
S.D. of Investment Rates	0.4	0.7	2.2	0.3	7.9

	Value					
Statistic	2010	2011	2012	2013	2014	2015
Inaction Rate (%)	28.4%	29.1%	30.9%	32.5%	36.5%	40.3%
Spike Rate (%)	26.3%	25.3%	21.2%	20.3%	17.3%	14.0%
Positive Investment Rate (%)	45.3%	45.7%	47.9%	47.2%	46.2%	45.7%
Average Investment Rate (%)	9.2%	9.5%	-1.3%	-2.4%	3.8%	-3.4%
S.D. of Investment Rates	0.6	0.4	5.7	6.4	0.7	3.0

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