Quantitative Easing as a Commitment Device in a Liquidity Trap

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November 1, 2022

Abstract

We show that when a central bank is not fully financially backed by the treasury and faces a solvency constraint, an increase in size or a change in the composition of its balance sheet (quantitative easing - QE) can serve as a commitment device in a liquidity trap scenario. In particular, when the short-term interest rate is at the zero lower bound, open market operations by the central bank that involve purchases of long-term bonds can help mitigate deflation and recession under a discretionary policy equilibrium. Using a simple endowment-economy model, we show that a change in the central bank balance sheet, which increases its size and duration, provides an incentive to the central bank to keep interest rates low in the future to avoid losses and satisfy its solvency constraints, approximating its full commitment policy. To test the validity of the novel mechanism, we incorporate a financially-independent central bank into a medium-scale DSGE model based on Smets and Wouters (2007), and calibrated it to replicate key features of the expansion of size and composition of the Federal Reserve's balance sheet in the post-2008 period. Simulating the future path of the federal funds rate at the exit of the 2008 crisis, we find that the financial stability of the Fed is at risk if monetary policy is conducted in a discretionary fashion. Moreover, assuming the Fed cannot receive a positive transfer from the U.S. Treasury in present value, we find that the programs QE 2 and QE 3 generated positive effects on inflation dynamics but a modest impact on the output gap.

*Mendes (Corresponding author): agalegomendes@worldbank.org. Date: November 2022. Keywords: central bank balance sheet, liquidity traps, quantitative easing, zero-lower bound, New Keynesian model.

1 Introduction

Since the financial crisis of 2008, many central banks have been forced to change their main policy tool away from the short-term interest rates. As the policy rates reached the zero lower bound (ZLB), they lost their suitability as instruments to stimulate the economy. In a sluggish recovery, there has been a search for alternative expansionary monetary policies. Central bank balance sheet expansion has been the most common choice. In the United States, the Federal Reserve (Fed) purchased a total of $1.75 trillion in agency debt, mortgage-backed securities (MBS) and Treasuries in the "QE1", followed by a second Treasury-only program of $600 billion in the fall of 2010. In September 2011, the Fed introduced QE3, increasing the amount of long-term bonds in its balance sheet. Other countries also followed similar strategies. In March 2009, the Bank of England (BoE) announced it would purchase a total of £75 billion of U.K. gilts, which, after subsequent increases, was expanded to £375 billion in July 2012. On 4 April 2013, the Bank of Japan (BoJ) announced a plan to purchase ¥7.5 trillion of bonds a month and double its monetary base. More recently, on January 2, 2015, the European Central Bank (ECB) announced monthly asset purchases of 60 billion euros, to be carried out until at least September 2016.

The stimulative role of QE has since been the focus of intensive debate. Empirically, many studies have demonstrated the effects of these programs on asset prices and interest rates.[1] However, the precise theoretical channel through which these programs affect real variables is unclear and is still under the scrutiny of the academic debate. Most recent mechanisms rely on segmented markets or other sources of financial frictions in order to generate real effects.[2] In this paper, we provide an alternative mechanism in which changes in central bank balance sheet have real effects. Specifically, when the central bank is restricted from incurring in huge financial losses, these programs act as a credible restriction on future monetary policy actions.

In addition, we show that central banks that face solvency constraints can use their balance sheets to mitigate the credibility issues that arise in optimal policy in a liquidity trap. In other words, a central bank that is restricted in the losses it can have is subject to a possible commitment mechanism: if its balance sheet is large or shows long enough duration, possible unfavorable asset price movements coming from interest rate hikes are going to be avoided, restricting upward shifts in the policy rates and leading to a credible higher inflation path. This commitment mechanism allows a discretionary central bank to approximate optimal commitment policies and provides a theoretical justification for the recent adoption of QE programs by several central banks as their short-term interest rates have reached the ZLB.

Identifying channels through which large purchase programs, such as QEs, have real effects is no trivial task. It has been well known since Wallace (1981) that changes in the size or the composition of the central bank's balance sheet has no effect on equilibrium allocations within the framework of general equilibrium models: in a representative agent-based model, a mere shuffling of assets between the central bank and the private sector should not change any asset price in the economy. Instead, macroeconomic theory prescribes a rather different policy in the

[1]See Gagnon et al. (2011), Hamilton and Wu (2012), Krishnamurthy and Vissing-Jorgensen (2011), and Williams (2011) and references therein.

[2]Among others, we refer to Gertler and Kiyotaki (2010), Gertler and Karadi (2013), Vayanos and Vila (2009), and Curdia and Woodford (2011)

liquidity trap scenario. As first noted by Krugman (1998), optimal monetary policy at the ZLB entails a commitment to keep short-term interest rates low for a long period in the future. This policy generates a higher level of expected real income and inflation in the future and provides the economy with the necessary incentives for greater real expenditure and larger price increases in the present. The problem also emphasized in Krugman (1998), is how to make low interest rates in the future credible: the central bank may renege ex post on its promises to pursue its goal of price stability. In fact, why would the central bank generate undesired inflation simply because of a binding constraint in the past?

Addressing this credibility problem, Woodford (2012) suggests the use of explicit statements by central banks about the outlook for future policy in addition to their announcements about the immediate policy actions that are in course. This type of policy, or *forward guidance*, is intended to facilitate the implementation of the optimal policy, as it makes it unambiguously clear that the central bank intends to maintain the benchmark rate at its lower bound for extended periods. Despite all the discussion of its effectiveness in practice, these announcements only constitute a commitment device if associated with costs of reneging (either moral or pecuniary).

Nakata (2018) finds that a central bank has the incentive to maintain the original announced path of low nominal interest rates, in order to build reputation, if contractionary shocks hit the economy frequently. If the central bank reneges on the promise of low policy rates, it will lose reputation and the private sector will not believe such promises in future recessions. However, it is possible that most central bankers see it differently, and fear that even a temporary inflation overshoot could undermine the central bank's reputation of pursuing price stability as their primary objective.

Instead of relying on hidden reneging costs, we design a mechanism through which the credibility problem in a liquidity trap scenario can be mitigated if central banks face solvency constraints. More specifically, this mechanism allows this type of central bank to commit to lower future interest rate through a large-scale purchase of long-term securities that creates an incentive not to raise interest rates in the future and thus, avoid losses on its balance sheet.

This result relies on two basic assumptions: (*i*) central banks are not financially backed by the treasury in all possible states of nature, and (*ii*) central banks cannot become insolvent. The first observation limits transfers between these authorities and adds a budget constraint to central banks. The second implies that central banks of this type cannot run unlimited losses.[3] We view these assumptions as a consequence of a self-imposed behavior motivated by the political embarrassment caused by large financial bail-outs. Together they provide an additional restriction to monetary policymakers: they cannot undertake actions that lead to excessive losses in their balance sheets. Accordingly, a current large-scale purchase of long-term securities can credibly lock the central bank into low interest rates in the future because interest rate hikes may threaten the central bank's solvency.[4]

Eggertsson (2006) was among one of the first works to analyze deflation as a credibility problem, and to formally think about a time-consistent implementation of the commitment solution in a liquidity trap. He proposes that a government can credibly commit to "being

[3]This is directly related to the literature that assumes balance sheet concerns on the part of the central bank, such as Sims (2004), Berriel and Bhattarai (2009), and Jeanne and Svensson (2007).

[4]For further reference on how interest rates affect central bank's balance sheets, see Hall and Reis (2015).

irresponsible" by increasing deficit during a liquidity trap. Inflation expectations would increase because higher nominal debt gives the government an incentive to inflate the real value of the debt away, instead of raising revenues through an increase in distortionary tax rates.

This work is also closely related to Jeanne and Svensson (2007) (JE07 hereafter). They showed that if central banks in small open economies have capital concerns, then it is possible to create a commitment mechanism that allows independent central banks to achieve a higher future price level through a present currency depreciation. This paper differs from JE07 in two important aspects. First, the commitment mechanism we designed does not rely on the small open economy assumption and hence is more suitable for the U.S. economy. Second, in JE07 capital concerns are modeled as ad-hoc preferences against low levels of capital that are difficult to assess and interpret in practice. Instead, we rely on the more realistic assumption that central banks will not undertake any actions that may undermine their capacity or independence to carry out monetary policy in the future. This is in line with Del Negro and Sims (2015), where low levels of capital can prevent a central bank from avoiding self-fulfilling hyper-inflationary equilibria, and Buiter (2008), where the scale of the recourse to seigniorage required to safeguard central bank solvency may undermine price stability. Bhattarai et al. (2015) focus on the implications of joint monetary and fiscal policy to a similar problem, while here we focus on the implications of limited losses of the central bank.

The practical consequences of our proposed theory of quantiative easing depends on the assumption that these programs actually threaten the financial stability of central banks. Hall and Reis (2015) investigate the resilience of the Federal Reserve to interest rate shocks. They find that on the exit of the 2008 financial crisis, the Fed makes large losses and the Treasury needs to provide for makeup reductions in payments from the bank to the Treasury in order to rebuild the bank's capital. Despite the large losses, they conclude that the Fed's financial stability is remote. We complement this branch of the literature by analyzing the financial stability of the Fed when monetary policy is conducted optimally under commitment and discretion. We first calibrate a simple exogenous-income model to the U.S. economy and the Fed's balance sheet, and find, much in line with Hall and Reis (2015), that although the Fed can make losses as high as \$225 billion on the exit of the crisis, it's financial stability is not in danger. We then setup a medium-scale DSGE model based on Smets and Wouters (2007) with more realistic assumptions about the structure of shocks hitting the U.S. economy, the Fed's balance sheet and the instutional arrangement betweent the Fed and the Treasury, and find that there is a significant threat to the Fed's financial stability, specially when monetary policy is conducted under discretion and remittances to the Treasury are based on the Fed's net interest income.

Finally, we use the DSGE model based on Smets and Wouters (2007) to analyze the consequences of the programs QE 2 and QE 3 to the equilibrium dynamics of the federal funds rate, inflation, and the output gap, assuming that the Fed is subject to solvency constraints. We find that the solvency constraint forces the Fed to deviate from the baseline optimal path of the federal funds rate, creating significant additional inflation but only mild impact on the output gap.

The rest of the paper is organized as follows. Section 2 describes a simple endowment economy model with a financially independent central bank that conducts monetary policy under discretion and commitment and is allowed to buy short and long-term government bonds. In section 3 we show how an increase in the size and composition of the central bank's balance sheet can serve as a commitment device to low interest rates in the future during a liquidity trap scenario. Section 4 discusses the results of the previous section and simulates the impact of QE on the Fed's balance sheet in the exogenous-income model. Section 5 describes the quantitative model based on Smets and Wouters (2007). Section 6 briefly discusses the empirical performance of the model. Section 7 uses the quantitative model to test the Fed's financial stability and the impact of QE on the federal funds rate, inflation and output gap. Section 8 concludes.

2 A Simple Endowment Economy Model

2.1 The Model Overview

In this section, we consider a one-good, representative agent economy. The household consumes and saves by buying riskless government bonds of different maturities. In this simple economy, we abstract from production and assume that consumption each period is restricted to an exogenous endowment process. The central bank is not fully financed by the Treasury and conducts monetary policy through a price-level targeting regime in which the policy rate is set to minimize a quadratic loss function of the price level. We introduce money in this economy by imposing a cash-in-advance constraint: in the beginning of each period, individuals trade cash for bonds, with net nominal interest rate i_t. Their consumption during the period is constrained by the cash with which they generate from this trading. We show in section 3 that the economy falls into a liquidity trap in period 1, with price level below the target, as a result of an unanticipated fall in expected endowment growth. The same scenario might arise in period 2, conditional on the realization of the endowment process.

2.2 The Household

The household's utility function is assumed to take the form,

$$U_t = \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \frac{C_{t+i}^{1-\sigma}}{1-\sigma}$$

where C_t is consumption in period t, $\mathbb{E}_t$ is the expectation operator conditional to available information in period t, β is the discount factor,and σ the coefficient of risk aversion. The household seeks to maximize utility subject to the following budget constraint,

$$P_tY_t+(1+i_{t-1})B_{t-1}^{hh}+\left(\frac{1+(1-\delta_b)Q_t}{Q_{t-1}}\right)B_{t-1}^{l,hh}+(1+i_{t-1}^m)M_{t-1} = P_tC_t+Z_t+B_t^{hh}+B_t^{l,hh}+M_t \quad (1)$$

where Y_t is a stochastic endowment process, M_t, B_t^h and $B_t^{l,h}$ are respectively the total of money, short and long-term claims on the government debt held by the household. The short-term bond costs 1 dollar in period t and pays nominal interest rate i_t in period $t+1$. We allow the central bank to pay nominal net interest rate, i_t^m, on the its monetary liabilities, M_t. The long-term bond costs Q_t dollars in period t and pays a 1-dollar coupon in period $t+1$. In $t+2$, the bond will pay a fraction $(1-\delta_b)$ of the coupon, $(1-\delta_b)^2$ in $t+3$, and so on. Lower values of δ_b correspond to portfolios with longer maturities. We add up bonds in terms of the amount of output they will pay in the current period so each period the bonds inherited from previous periods shrink by the factor $1-\delta_b$, and $1/\delta_b$ is the average maturity of the portfolio holdings. Each period the government collects lump-sum tax Z_t, which is denoted in real terms.

2.3 The Endowment Process

As mentioned before, there is no production and each period consumption is restricted to the exogenous income process, Y_t. We assume that, from indefinitely long before period 1, the agent has been receiving a certain income $y^* e^{\bar{y}}$. In period 1, however, the agent is informed that from period 2 onwards income will follow the process described by expression (2), and that the resolution of uncertainty on this process will become available information to the agent only in period 2,

$$(Y_1, Y_2, Y_3, ..., Y_{N-1}, Y_N, Y_{N+1}, ...) = \begin{cases} (y^* e^{\bar{y}}, y^*, y^*, ..., y^*, y^*, y^*, ...) & \text{with probability } 1-\mu \\ (y^* e^{\bar{y}}, y^*, y^* e^{\underline{y}}, ..., y^* e^{\underline{y}}, y^*, y^*, ...) & \text{with probability } \mu \end{cases} \quad (2)$$

where y^* is the income of the upcoming steady state, $\bar{y} > 0$ and $\underline{y} < 0$. In section 3 we show that in period 1 the unexpected fall in income growth pushes the economy into a liquidity trap as a result of the interaction between the agent's excess savings and the ZLB. This liquid trap scenario continues in period 2 with probability μ in the low-income realization of process (2), or reverts with probability $1-\mu$ in the high-income realization of (2).

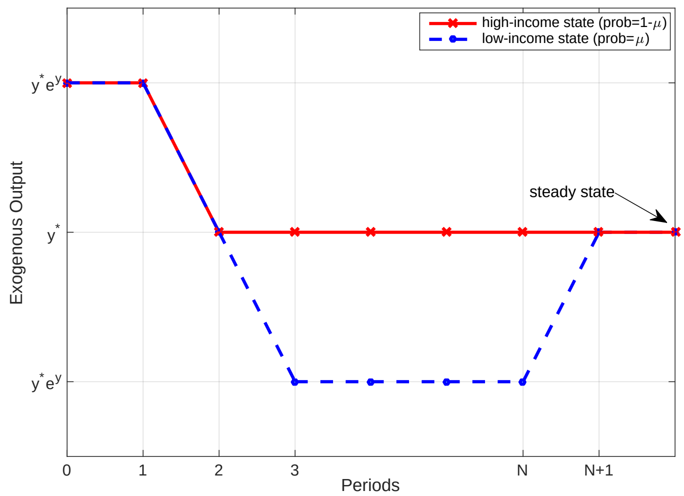


Figure 1: **A Simple Endowment Economy Model: The Endowment Process.**

Figure (1) depicts the endowment process. Note that after N periods, the income returns to steady-state level, y^*, independently of the realization of process (2), so we have a well defined non-stochastic steady state.

2.4 The Public Sector

2.4.1 The Central Bank

Increasing literature points to the fact that central banks are not fully financed by the treasury in all contingencies. This is more evident in cases where the central bank faces risks of unusually large losses in its balance sheet. Following these concerns, we introduce a central bank that is not fully financially backed by the treasury. Since the central bank cannot rely on the treasury for all its financial needs, it is subject to a period-by-period budget constraint,

$$B_t^{cb} + B_t^{cb,l} + D_t = (1 + i_{t-1})B_{t-1}^{cb} + \left(\frac{1 + (1 - \delta_b)Q_t}{Q_{t-1}}\right) B_{t-1}^{cb,l} + M_t - (1 + i_{t-1}^m)M_{t-1}$$

where B_{t-1}^{cb} and $B_{t-1}^{l,cb}$ denote the dollar-denominated stock of short and long-term government bonds held by the central bank in period t, respectively. The variable M_t is the outstanding central bank monetary liabilities and D_t denotes dividends paid to the treasury. All measured in nominal US dollars.

One important assumption is that, while considering its budget constraint, all assets of the central bank are marked-to-market. This is a trivially appropriate assumption for modeling the ECB or the Bank of England, which are obliged by law to report this type of pricing. However, if one considers the Fed, this assumption is debatable. In principle, the Fed *can* and actually has adopted historical prices in calculating gains or losses in his balance sheet. We argue that if one is worried about the political implications of a possible recapitalization or decrease in revenues from the Fed to the U.S. Treasury, a large gap between historical and market valuations would be an embarrassment for the Fed. So, even without reporting it, the Fed would care about marked-to-market gains and losses in its balance sheet.

In this paper we adopt the view that a central bank endorses the mark-to-market accounting, assessing the value of its portfolio at market prices under any circumstance. In this case, net income is defined as net interest income plus capital gain. The first being the net return on the short-term bond portfolio, plus coupon less revaluation on the long-term bond portfolio, less interest cost on reserves,

$$NI_t = \underbrace{i_{t-1}B_{t-1}^{cb} + \left(\frac{1 - \delta_b Q_{t-1}}{Q_{t-1}}\right) B_{t-1}^{cb,l} - i_{t-1}^m M_{t-1}}_{\text{Nominal Net Interest Income } (\equiv NII_t)} + \underbrace{\left(\frac{Q_t^b}{Q_{t-1}^b} - 1\right) B_{t-1}^{cb,l}}_{\text{Nominal Capital Gains or Losses}} \quad (3)$$

Net income is an important concept because it underlies the size of remittances foreseen in the contract between the fiscal and the monetary authorities. Usually, central banks transfer a share of its net income to the treasury in terms of seigniorage revenues. We model this agreement by assuming the following dividend rule,

$$D_t = NI_t$$

The rule D_t is key in this paper. It is important to note that these transfers paid by the central bank to the treasury could be negative. Such transfer payment from the treasury to the central bank can be viewed as the mechanism through which the treasury can inject capital into the central bank, i.e., transfer resources to the central bank in order to recapitalize it.

In normal times, the assets and liabilities of a central bank are nearly riskless and net income is usually positive. When the central bank holds other types of assets, especially private debt, and assets subject to nominal losses, net income could be negative with significant probability. Negative net income requires fiscal backing to the central bank. The act of capitalizing the central bank would have to be approved by fiscal authorities, subject to the underlying political process.

Even if feasible in economic terms, a fiscal bailout of the central bank is not necessarily politically implementable. In many occasions, the taxpayer is simply not willing to abdicate on real resources (and, thus, consumption), in order to support the central bank's balance sheet. An interesting example is the ECB, where it is not clear how losses would be split among different fiscal authorities. We include these considerations in the model by assuming the following dividend rule between the central bank and the treasury,

$$D_t = \begin{cases} NI_t & \text{if } NI_t \geq -\xi \\ 0 & \text{otherwise} \end{cases}$$

where $\xi > 0$. Positive net income is transferred to the Treasury as dividends on seignorage. We allow for a limited degree of fiscal backing. The treasury is allowed to cover central bank's losses if it sits below a predefined threshold. If $NI_t < -\xi$, recapitalization is blocked by fiscal authorities.

The central bank's net worth, NW_t, is defined as the excess of the value of the bond portfolio marked to market over the size of the monetary liabilities, $NW_t = B_t^{cb} + B_t^{cb,l} - M_t$. Note that we can rewrite the central bank's balance sheet recursively as

$$NW_t = NW_{t-1} + NI_t - D_t \tag{4}$$

Central bank insolvency is an issue of considerable controversy since the vast majority of its liabilities is irredeemable. As pointed out by Sims (2004), while a central bank can always pay all its home-currency denominated expenses (financial or operational) through the issuance of base money it may not be optimal or even acceptable: it may generate inadmissible high rates of inflation. In addition, there are limits to the amount of real resources the central bank can appropriate by increasing the issuance of nominal base money.[5] Hence, despite the central bank's special ability to issue not just non-interest-bearing but also irredeemable liabilities, central bank's solvency is questioned if its capital falls below some specified level. According to Hall and Reis (2015), a central bank is independent as long as it adheres to its dividend rule and the rule does not imply explosive reserve growth. These authors take interest rates and inflation as given and study the implications of an economic recovery from the Great Recession to the financial stability of the central bank. We take the opposite approach: *we assume that*

[5] See Buiter (2008).

central banks remain solvent and study the implications for optimal monetary policy. We rule out insolvency in the model by imposing a lower bound on the central bank's net worth[6],

$$NW_t \geq -\phi \tag{5}$$

where the parameter ϕ can be interpreted as a physical limit imposed by fiscal authorities or a self-imposed restriction in light of the uncertainties of a bail-out. We take a similar approach to Hall and Reis (2015) and assume that ϕ represents the present value of seignorage, so that the central bank does not need to receive a positive transfer from the fiscal authority in present value. Expression (5) implies that policymakers are prohibited to undertake policies proposals that lead the central bank to insolvency or that severely compromise the financial status of the bank.

This solvency constraint is related to the literature that assumes balance-sheet concerns on the part of the central bank. Isard (1994) presented a model of currency crises in which the central bank cares about the value of its foreign exchange reserves. More recently, Jeanne and Svensson (2007) assumed that the central bank has an objective function with a fixed loss suffered if the capital of the central bank falls below a critical level. Berriel and Bhattarai (2009) modeled balance sheet concerns by including a target for real capital in the central bank's loss function. These works assume ad-hoc preferences of the central bank against negative or even low levels of capital. Note that the solvency constraint (5) simply prevents the central banker from taking certain policy actions in certain situations, and says nothing about central bankers' preferences about capital adequacy. This is in line with Del Negro and Sims (2015), where low levels of capital may prevent a central bank from avoiding self-fulfilling hyperinflationary equilibria.

It remains to specify the objective of monetary policy and how the central bank manages different instruments to achieve its goals. We assume that the central bank has an objective function corresponding to a price-level targeting regime. The central bank's intertemporal loss function can be written as

$$L_t = \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i (log(P_{t+i}) - 1)^2$$

where 1 is the targeted price level in the economy. The price level target is a simplification that allows for a simple analytical solution of the model. The quantitative model of section (5), substitutes this ad-hoc assumption by a standard loss function in terms of inflation and output gap.

The monetary authority has three instruments to achieve its goal of price stability: the policy rate (i_t), interest paid on reserves (IOR) (i_t^m) and quantitative easing ($B_t^{cb,l}$). We assume that in each period, the central bank sets the IOR equal to the policy rate,

[6]Note that we are imposing this solvency restriction in nominal terms. This simplifying assumption is dropped in the quantitative model when we take this restriction in real terms.

$$i_t = i_t^m \tag{6}$$

This is a convenient assumption since it implies zero net interest income, allowing for a simple analytical solution of the model. Moreover, (6) captures an important feature of the new way that many central banks have been operating around the world since the financial crisis. For example, in the US, the Emergency Economic Stabilization Act of 2008 allowed the Fed to begin paying interest on excess reserve balances ("IOER") as well as required reserves.

In the model, quantitative easing is very simple: the central bank implements a constant target for holdings of long-term government bonds in its balance sheet,

$$B_t^{cb,l} = B_*^l \tag{7}$$

where B_*^l is not under the control of policymakers but rather chosen to match the size of Fed's balance sheet observed in the data. Note that the purpose of this work is not to investigate the optimality of QE but rather to take it as given and assess its implications to conventional monetary policy when the central bank is fiscally constrained.

We assume that the central bank implements adjustments to the policy rate through conventional open-market operations with short-term bonds. In this case, holdings of short-term bonds, B_t^{cb}, is determined in the balance sheet so that the central bank supplies the desired liquidity required by households,

$$B_t^{cb} = NW_t + M_t - B_t^{cb,l} \tag{8}$$

Monetary policy is also subject to the zero lower bound (ZLB) on the short-term nominal interest rate,

$$i_t \geq 0 \tag{9}$$

2.4.2 The treasury and fiscal policy

For simplicity we abstract from government expenditure and assume that the fiscal authority receives dividends from the central bank, D_t, and collects lump-sum taxes, Z_t. The treasury's budget constraint can be written as,

$$D_t + Z_t + B_t^{hh} + B_t^{cb} + B_t^{cb,l} + B_t^{hh,l} = (1 + i_{t-1}) \left(B_{t-1}^{cb} + B_{t-1}^{hh} \right) + \left(\frac{1 + (1 - \delta_b) Q_{t-1}}{Q_{t-1}} \right) \left(B_{t-1}^{l,hh} + B_{t-1}^{l,cb} \right) \tag{10}$$

We specify fiscal policy in terms of a rule that determines the evolution of lump-sum taxes responding to contemporaneous level of total real government debt,

$$\frac{Z_t}{P_t} = exp\left\{\phi_z\left(\frac{B^{hh}_{t-1} + B_{t-1} + B^{hh,s}_{t-1} + B^s_{t-1}}{P_t}\right)\right\} \tag{11}$$

We choose the parameter ϕ_z so that fiscal policy is "Ricardian": lump-sum taxes adjust sufficiently fast to ensure that the trajectory of government debt is non-explosive, regardless the path of the price level.[7]

2.5 Equilibrium

Lemma 1 *Consider a linear rational expectations model formed by the system of equations* (1)-(11) *linearized around the zero-inflation steady state. If the fiscal authority is not allowed to back the central bank in case of insolvency, i.e. $\xi = \phi$, we can reduce the model to a 3-equations linear system*

$$\hat{y}_t = \hat{y}_{t+1|t} - \sigma^{-1}\left(i_t - (\hat{p}_{t+1|t} - \hat{p}_t) - \rho\right) \tag{12}$$

$$\hat{q}_t = \beta(1-\delta_b)\hat{q}_{t+1|t} - (i_t - \rho) \tag{13}$$

$$\tilde{ni}_t = b^l_*\left(\hat{q}_t - (1+\rho-\delta_b)\hat{q}_{t-1}\right) \tag{14}$$

together with the non-linear constraints

$$\tilde{ni}_t \geq -(\phi + ni_*) \tag{15}$$

$$i_t \geq 0 \tag{16}$$

and the log-linearized endowment process,

$$(\hat{y}_1, \hat{y}_2, \hat{y}_3, ..., \hat{y}_{N-1}, \hat{y}_N, \hat{y}_{N+1}, ...) = \begin{cases} (\bar{y}, 0, 0, ...) & \text{with probability } 1-\mu \\ (\bar{y}, 0, \underline{y}, ..., \underline{y}, 0, 0, ...) & \text{with probability } \mu \end{cases} \tag{17}$$

where stared variables denote steady state levels, hatted variables denote percent deviation from steady state $\left(\hat{x} = \frac{x_t - x_*}{x_*}\right)$, tilded variables denote deviations from steady state as a share of steady state GDP $\left(\tilde{x} = \frac{x_t - x_*}{y_*}\right)$ and $\theta_b \equiv \beta(1-\delta_b)$. A list of all linearized equations and a proof for the proposition are provided in the technical appendix.

Notation *Let s_i denote the nodes of the exogenous income process* (17)*, for $i \in \{l, h\}$. Where "h" indicates the realization of the high-income state and "l" represents the low-income state.*

[7] The terminology "Ricardian" fiscal policy is borrowed from Woodford (2001). "Passive" fiscal policy has equivalent interpretation, as in Leeper (1991). Note that model is not overdetermined because the treasury's budget constraint is a mirror of the household's budget constrain, so that B^{hh}_t will always adjust to close the treasury's budget regardless of Z_t .

Definition 1 *We define a discretion equilibrium as a sequence for prices* $\{\hat{p}_t, i_t, \hat{q}_t\}$ *and quantities* $\{\tilde{ni}_t, \hat{y}_t\}$ *as functions of the stochastic variable* $\{s_i\}$ *and the endogenous state* $\{\hat{q}_{t-1}\}$ *such that the central bank's intertemporal loss function,* L_t*, is minimized every period subject to* (12)-(17) *when the central bank cannot commit to future policies.*

Definition 2 *We define a commitment equilibrium as a sequence for prices* $\{\hat{p}_t, i_t, \hat{q}_t\}$ *and quantities* $\{\tilde{ni}_t, \hat{y}_t\}$ *as functions of the stochastic process* $\{s_i\}$ *and the endogenous state* $\{\hat{q}_{t-1}\}$ *such that the central bank's intertemporal loss function,* L_t*, is minimized in period 1 subject to* (12)-(17) *when the central bank can commit to future policies.*

3 Fiscally Constrained Central Bank and Quantitative Easing

In this section, we assume that $\phi = \xi < \infty$ so that the solvency constraint is a relevant restriction to equilibrium. We show that a long-term bond purchase program (or QE) can help mitigating deflation in a discretionary equilibrium. This is because a change in the size and composition of the balance sheet provides the central bank with the incentive to keep low interest rates in period 2 and avoid large financial losses.

More specifically, we show that for any given loss limit, ϕ, there is a positive level of long-term bond holdings, $b_*^{cb,l}$, such that, if the zero lower bound is binding in period 1 then the solvency constraint is binding, at least, in the high-endowment state of period 2. The solvency constraint prevents an interest rate rike and alleviates deflationary pressures in period 1.

Definition 3 *Let* $\tilde{\phi}_b \equiv \frac{\phi}{(1+\rho-\delta_b)b_*^l}$. *The fraction* $\tilde{\phi}_b$ *can be interpreted as a measure of risk. It denotes the largest percentage fall in the market value of long-term bonds in one period, conditional on the central bank remaining solvent in that period.*

Note that both an increase in the size or duration of the balance sheet will decrease $\tilde{\phi}_b$. A lower $\tilde{\phi}_b$ means that the central bank balance sheet is less resilient to the volatility of long-term bond prices.

Next, we make three assumptions about the parameter space. Assumptions (i) and (ii) ensure that the endowment process (17) pushes the economy against the ZLB in period 1 and in the low-income state of period 2. Condition (iii) guarantees that the solvency constraint is tight enough to restrict an interest rate hike in the high-income state of period 2, but not so tight that the central bank is insolvent regardless the choice of the policy rate.

A1 Assume (i) $\underline{y} < -\rho\sigma^{-1}$, (ii) $\bar{y} \geq \rho\sigma^{-1}(2+\beta) + (1+\beta)\mu\underline{y}$ *and* (iii) $\rho \leq \tilde{\phi}_b \leq \rho(1+\mu\theta_p)$

3.1 Solving the Model under Discretionary Monetary Policy

In this endowment economy, it is intuitive to think in terms of an equilibrium real interest rate, which will be in effect no matter the behavior of nominal prices. In "normal" times, when expected income growth is non-negative, the equilibrium real interest rate is positive and policymakers can readily implement the policy rate that is consistent with the price-level target if this policy does not threaten the solvency of the central bank.

Under the specific assumptions of this model, the equilibrium interest rates will be positive in the high-income state of period 2 and from period 3 onwards,

$$r_t^n(s_i) \equiv i_t - (\hat{p}_{t+1|t}^i - \hat{p}_t^i) = \rho > 0 \qquad \text{for all } 3 \leq t < N, \text{ and } t = 2 \text{ if } s_i = s_h,$$

where $r_t^n(s_i)$ denotes the natural interest rate in period t when state i occurred. If the solvency does not bind, one can immediately guess at the solution: the central bank sets the nominal interest rate equal to ρ and the price level immediately converges to the target. Condition $\tilde{\phi}_b \geq \rho$ assures that the loss limit is large enough to allow the central bank to pursue this policy scheme from period 3 onwards, independent of the realization of the income process.

Lemma 2 *Assume A1, $\phi = \xi$ and $N \to \infty$. The equilibrium under dicretionary monetary policy is characterized by $\{\hat{p}_t(s_i), i_t(s_i), \hat{q}_t(s_i)\} = \{0, \rho, 0\}$ for $t \geq 3$ and $s^i = \{s^l, s^h\}$. A proof is provided in the technical appendix.*

Low-Income State of the Second Period. Condition (i) brings about a large fall in expected income growth that pushes the natural rate of interest into negative territory, and the central bank faces a credibility problem, as in Krugman (1998),

$$r_2^n(s^l) = i_2(s^l) - (\underbrace{\hat{p}_{3|2}(s^l)}_{=0} - \hat{p}_2(s^l)) = \rho + \sigma \underline{y} < 0$$

Because of the ZLB, the only way the economy can achieve negative real interest rates is by generating inflation expectations. Because the central bank cannot commit to a higher target in period 3, the price level will have to fall to clear the market in period 2. Note that, as in Wallace (1981), the price level will fall regardless of the current money supply because any excess money will simply be kept rather than spent. This happens because once the nominal rate reaches zero, money and bonds become perfect substitutes and no matter how much liquidity the central bank injects in the economy, it can no longer affect asset prices. Lemma (3) summarizes the outcome in this state,

Lemma 3 *Assume A1, $\phi = \xi$ and $N \to \infty$. The equilibrium under discretion in the low-income state of period 2 is characterized by*

$$\{\hat{p}_2(\hat{q}_1, s_l), i_2(\hat{q}_1, s_l), \hat{q}_2(\hat{q}_1, s_l)\} = \{\rho + \sigma \underline{y}, 0, \rho\} \tag{18}$$

A proof is provided in the technical appendix.

High-Income State of the Second Period. In the high-income state of period 2, the central bank faces a positive natural real interest rate, ρ, and hence can achieve the target by setting the policy rate to ρ,

$$r_2^n(\hat{q}_1, s_h) = i_2(\hat{q}_1, s_h) - (\underbrace{\hat{p}_{3|2}(\hat{q}_1, s_h)}_{=0} - \hat{p}_2(\hat{q}_1, s_h)) = \rho > 0$$

The key difference in this state is that the solvency constraint might bind and prevent the central bank from choosing the optimal discretionary policy. The price of the long-term bond in the previous period, $\hat{q}_1$, plays an important role in determining the equilibrium in this state. Recall that $\tilde{\phi}_b$ is the largest fall in the price of long-term bonds that the central bank can absorb while still solvent. Hence, if $\hat{q}_1 > \tilde{\phi}_b$, the central bank will be forced to slow down the contractionary cycle of monetary policy to avoid insolvency. Lemma (4) summarizes,

Lemma 4 *Assume A1, $\phi = \xi$ and $N \to \infty$. The equilibrium under discretionary monetary policy in the high-income state of the second period is characterized by*

$$i_2(\hat{q}_1, s_h) = \begin{cases} \rho & \text{if} \quad \hat{q}_1 \leq \tilde{\phi}_b \\ \rho - (1+\rho-\delta_b)(\hat{q}_1 - \tilde{\phi}_b) & \text{if} \quad \tilde{\phi}_b < \hat{q}_1 \leq \tilde{\phi}_b + \frac{\rho}{1+\rho-\delta_b} \\ 0 & \text{if} \quad \tilde{\phi}_b + \frac{\rho}{1+\rho-\delta_b} < \hat{q}_1 \end{cases}$$

$$\hat{p}_2(q_1, s_h) = \hat{q}_2(q_1, s^h) = \rho - i_2(q_1, s^h) \tag{19}$$

$$\tilde{nw}_2(q_1, s_h) = \begin{cases} -\left(\frac{\phi}{\tilde{\phi}_b}\right)\hat{q}_1 & \text{if} \quad \hat{q}_1 \leq \tilde{\phi}_b \\ -\phi & \text{if} \quad \tilde{\phi}_b < \hat{q}_1 \leq \tilde{\phi}_b + \frac{\rho}{1+\rho-\delta_b} \end{cases}$$

A proof is provided in the technical appendix.

When the $\hat{q}_1 > \tilde{\phi}_b$, the solency constraint binds and prevents the central bank from raising interest rates, resulting in an undesired high price level, $\hat{p}_2(q_1, s_h) > 0$. Moreover, in the range between $\tilde{\phi}_b$ and $\tilde{\phi}_b + \frac{\rho}{1+\rho-\delta_b}$, the equilibrium price level increases with $\hat{q}_1$ at the rate $1 + \rho - \delta_b$.

First Period. In the first period the central bank chooses the policy rate i_1 to minimize the intertemporal loss function, L_1, taking into account that this decision affects $\hat{q}_1$, and hence expectations about next periods's price level. The private sector condition its expectations to $\hat{q}_1$, using expressions (18) and (19) and the probability distribution of the endowment process,

$$\hat{p}_{2|1}(q_1) = \mu\hat{p}_2(\hat{q}_1, s_l) + (1-\mu)\hat{p}_2(\hat{q}_1, s_h) \tag{20}$$

$$\hat{q}_{2|1}(\hat{q}_1) = \mu\hat{q}_2(\hat{q}_1, s_l) + (1-\mu)\hat{q}_2(q_1, s_h) \tag{21}$$

The problem faced by the central bank is

$$
\begin{aligned}
\underset{\{i_1 \geq 0\}}{min} \quad & \frac{1}{2}\left[\hat{p}_1^2 + \beta\left(\hat{p}_{2|1}(\hat{q}_1)\right)^2\right] \\
\text{s.t.} \quad & r_1^n = i_1 - \left(\hat{p}_{2|1}(\hat{q}_1) - \hat{p}_1\right) = \rho - \sigma\bar{y} \\
& \hat{q}_1 = \theta_q \hat{q}_{2|1}(\hat{q}_1) - (i_1 - \rho) \\
& \tilde{ni}_1 = b_*^l\left(\hat{q}_1 - (1+\rho-\delta_b)\hat{q}_0\right) \geq -\phi \\
& \text{(18) - (21) given } \hat{q}_0 = 0
\end{aligned}
$$

Proposition 1 *Assume A1, $\phi = \xi$ and $N \to \infty$. The equilibrium under discretionary monetary policy in the first period is characterized by*

$$
\begin{aligned}
i_1 &= 0 \\
\hat{q}_1 &= \rho\left(\frac{(1+\mu\theta_q)}{1-(1-\mu)(1+\rho-\delta_b)\theta_q}\right) - \theta_q \Xi \tilde{\phi}_b \\
\hat{p}_1 &= \underbrace{\rho - \sigma\bar{y} + \mu(\rho + \sigma \underline{y})}_{\text{unconstrained discretion}} + \underbrace{(1-\mu)\Xi\left(\rho(1+\mu\theta_q) - \tilde{\phi}_b\right)}_{\text{QE effect}}
\end{aligned} \tag{22}
$$

where $\Xi \equiv \left(\frac{(1+\rho-\delta_b)}{1-(1-\mu)(1+\rho-\delta_b)\theta_q}\right)$. A proof is provided in the technical appendix.

As highlighted by expression (22), the equilibrium price level in period 1 is the sum of the baseline price level (that would prevail in the absence of the solvency constraint) with the QE effect. The QE effect is always non-negative and depends on the size and duration of the central bank's balance sheet.[8] When the central bank expands its balance sheet through an increase in b_*^l, the parameter $\tilde{\phi}_b$ shrinks, tightening the solvency constraint in the high-income state of period 2, and boosting the price level in period 1 when the economy is stuck at the ZLB. Moreover, the marginal effect of QE depends on the average duration of the balance sheet: Ξ increases with $1/\delta_b$, and $\hat{p}_1$ collapses to the baseline price level as $1/\delta_b \to 1$.

These results provide a theoretical support for the use of non-conventional monetary policies, such as QE, by central banks to achieve price stability when the short-term policy rate is up against the ZLB.

4 Discussion

In this section, we use the simple model developed in sections 2 - 3 to analyze the effects of unconventional monetary policy on a fiscally constrained central bank since the financial crisis in 2008. We calibrate the model to resemble basic features of the US economy and the Fed's balance sheet in the period 2009-2013 when the bank implemented three rounds of large-scale asset pruchase programs: QE1, QE2, and QE3. We then use the calibrated model to evaluate the consequences of these programs to the financial stability of the Fed, and the dynamics of inflation.

[8] The non-negativity of the QE effect depends on condition (*iii*) of A1.

Calibration. We follow Hall and Reis (2015) and use data in the annual report of the Federal Reserve on the value and maturity of the U.S. Treasury securities it holds, to calculate the value-weighted average maturity of the Fed's financial assets. Between 2009 and 2013, the average maturity of the Fed's portfolio was 7.8 years. We measure the holdings of long-term bonds, $b_*^{cb,l}$, as the total U.S. Treasury and agency securities held by the Fed as a share of GDP. We assume that the Fed holds long-term bonds in an amount equal to 15 percent of annual GDP throughout the period in consideration. We choose the loss limit, ϕ, to be 2 percent of annual GDP, which is the present value of seignorage revenues used in Hall and Reis (2015).

Hall and Reis (2015) estimate the exit rate of the crisis to be 20 percent per year. Therefore, a crisis has an expected life of 5 years. To replicate the crisis duration in the 3-period structure of our model, we choose $\beta = 0.94$ and $\mu = 0.65$. Thus, steady-state annual real interest rate is 2 percent if we interpret each period of the model as being 3 years long. In this case, in period 1 the economy faces a 3-year crisis with low income and zero interest rates. In period 2, either the liquidity trap persists for another 3 years with 65% probability or the economy recovers with 35% chance. We choose $\bar{y}$ and $\underline{y}$ so that the expected accumulated forgone income is 25% of annual GDP, which is the accumulated gap between potential and actual GDP observed in the data during the period 2009-2013 (see figure 4 in section (9) for details).[9] Table 1 in section (10) summarizes the model's calibration and steady state.

The Effects of Quantitative Easing. The left-hand side of figure 2 plots the state-contingent equilibrium paths for the price level, interest rate, long-term bond price and the Fed's net worth. The blue dashed line shows the evolution of these variables in the high-income state of the income process and the red line represents the low-income state. For the sake of comparision with well known results in the literature, we plot on the right-hand side of figure 2 the equilibrium paths of the nominal interest rate and the price level under the baseline commitment (upper panel) and discretion (lower panel), both without QE ($b_*^l = 0$).To simplify the comparison, we asume that under commitment the Fed has the ability to commit to a price level target only in period 2.[10]

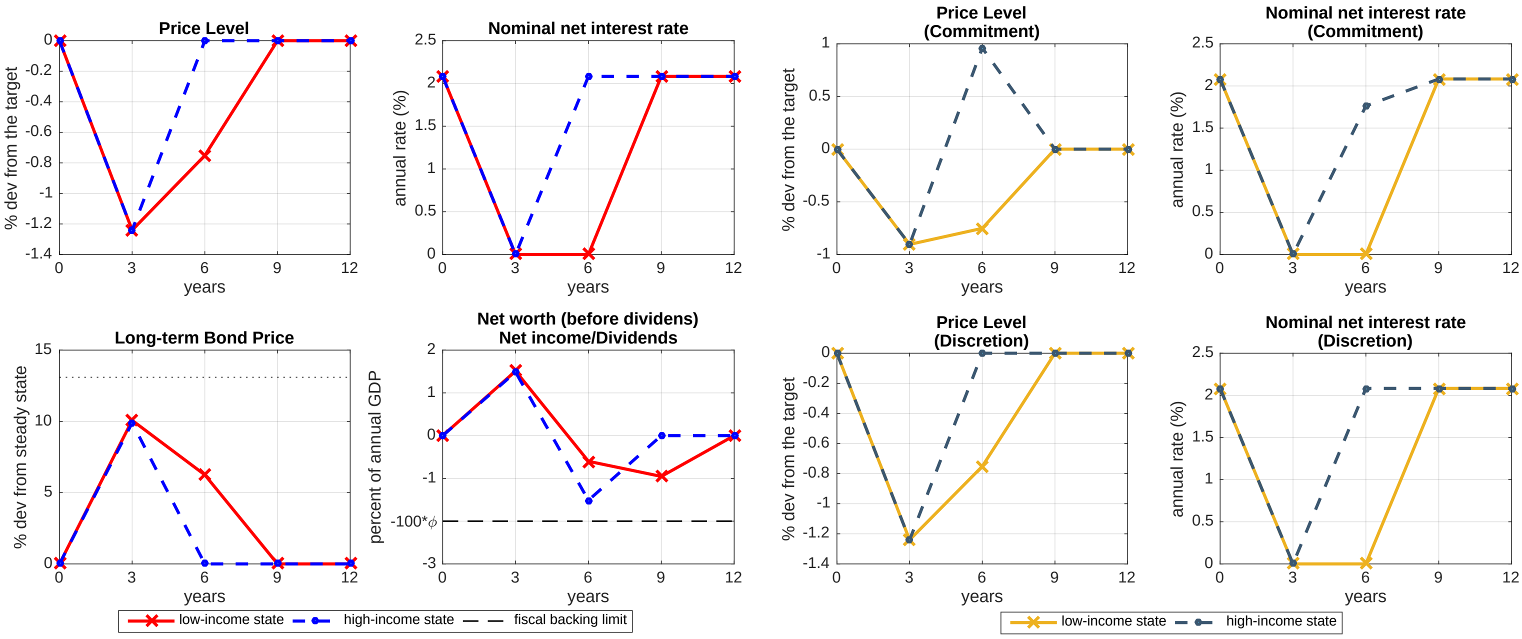
When the economy enters the crisis state in period 1, the central bank lowers the short-term interest rate to zero to counteract the deflationary pressures coming from the household's attempt to smooth consumption. As a result of lower, current, and expected interest rates, the bond price increases 10% and the Fed makes a large profit from its holdings of long-term bonds, 1.5% of GDP ($225 billion). In compliance with the dividend rule (**??**), the Fed remits the current net income to the Treasury. In the high-income state of the second period, economic conditions improve and the Fed raises interest rates to 2% a year, the long-run level. The bond price plummets from the 10% peak in the first period and the bank suffers a large loss of 1.5% of GDP (symmetrical to the gain in period 1). However, the loss *is not large enough* to put the financial stability of the Fed at risk. The present value of seignorage is estimated to be 2% of GDP and hence the Fed remains solvent. This result is in line with Hall and Reis (2015).[11]

[9]The accumulated gap between potential and actual during the crisis in the model is given by $\bar{y} + \mu(\bar{y} - \underline{y})$

[10]As aforementioned, without QE, assets held by the Central Bank are riskless and the solvency constraint can be disregarded.

[11]Although this model does not account for many important features of the crisis and the operating system of the Fed, our results seem to capture well the dynamics of the Fed's balance sheet estimated by Hall and Reis (2015), using a much more detailed approach. Hall and Reis (2015) find that the Fed would earn around $ 110

Because the solvency constraint is not active, the private sector expects a contractionary monetary policy when the crisis is over, resulting in a 1.2% fall in the price level in the first period. As illustrated in the upper panel on the RHS of figure 2, the optimal policy involves a commitment to a higher price-level target and lower nominal interest rate in the high-income state of the second period. The lower panel shows that the conventional discretionary equilibrium without QE is identical to the outcome with QE, and hence QE had no effect whatsoever.[12]



(a) Discretion & Baseline Calibration **(b) Discretion and Commitment & no QE**

Figure 2: **Inflation dynamics and the financial stability of the Fed in a liquidity trap.** Benchmark calibration (panel a): state-contingent path of the short-term nominal interest rate, price-level, price of long-term bonds and the Fed's net worth under discretion. Alternative calibration without QE ($b^l_* = 0$) (panel b): optimal commitment (upper panel) and discretion (lower panel).

Optimal Quantitative Easing. Is there a specific size (or duration) of the Fed's balance sheet that, at least theoretically, policymakers could implement the commitment equilibrium in a discretionary and time-consistent setup? Figure 3 shows the state-contingent path of key variables when the Fed holds long-term bonds in an amount equal to 21 percent of annual GDP. The equilibrium paths of short-term interest rates and price level replicate exactly the optimal commitment equilibrium. The solvency constraint prevents the central bank from raising the interest rate in the high-income state of period 2. As a result, the price-level overshoots the

billion in profits at the outset of the crisis, and loose \$220 billion when the economy shifts from the crisis state to normal times. They also conclude that, although theoretically plausible, it is unlikely that the Fed faces a real risk of insolvency.

[12] Eggertsson and Woodford (2003) and Billi (2013) argue that discretionary monetary policy under a price level targeting can approximate the optimal commitment solution. If the price-level target is not reached because of the ZLB, the central bank increases its target for the next period. This, in turn, increases inflation expectations further in the liquidity trap, which reduces the real interest rate, stimulating the economy. However, note that the discretionary policy creates some inflation in the high-income state of the second period, but much less than what is desirable under the fully optimal commitment.

target, providing the desired inflation expectations to lower real interest rates in period 1, when monetary policy is stuck at the ZLB, mitigating deflation in that period.

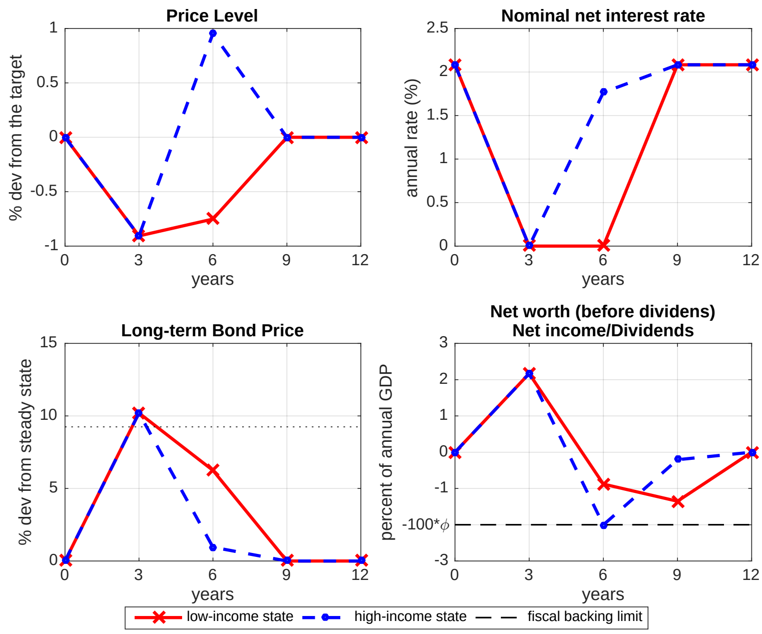


Figure 3: **Optimal Quantitative Easing:** state-contingent path of the short-term nominal interest rate and the price-level under discretion when the Fed holds 21 percent of annual GDP in long-term bonds.

The conclusion of this section is that the unconventional policies that resulted in the Fed borrowing 2.25 trillions of dollars (15% of GDP) from commercial banks to buy risky long-term Treasury securities did not threaten the solvency of the bank. However, a larger version of QE, in which the Fed purchases long-term bonds in an amount of equal to 21 % of annual GDP, would have had supported the optimal commitment outcome in a discretionary setup.

Although the Fed remains solvent when it raises interest rates on the exit of the crisis, it suffers a very large capital loss. It raises the questions of whether this result is robust to a richer environment with endogenous production, sticky prices and wages, capital accumulation and a more realistic structure of exogenous shocks. Moreover, we would like to see if these results go through in a model with inflation targeting (instead of price-level targeting), variables related to the balance sheet expressed in real terms and more realistic assumptions about the financial arrangement between the Fed and the Treasury. In the next section, we address these concerns using a standard medium-scale DSGE model based on Smets and Wouters (2007).

5 Quantitative Model

The model used in this section is based on the New Keynesian DSGE model developed in Smets and Wouters (2007) (SW07 hereafter). The choice of the SW07 model is appropriate since it is widely disseminated as a standard framework for quantitative policy analysis for the

U.S. economy. We introduce three main innovations in the baseline model in order to analyze the impact of QE on the balance sheet of the central bank and its consequences to inflation, employment, and output. First, we impose a ZLB. Second, we substitute the usual Taylor rule with the assumption that the policy rate is set optimally under commitment and discretion. Third, we add variables and equations that are related to the balance sheet of the central bank, and introduce two types of solvency constraints.

5.1 Main Features of the Smets and Wouters 2007 DSGE Model

Smets and Wouters (2007) use US data on real wages, hours worked, real GDP, consumption, investment, prices and the short-term nominal interest rate to estimate a medium-scale DSGE for the US economy, covering the period 1966:Q1 - 2004:Q4. The dataset allows the introduction of seven types of structural shocks: productivity, risk-premium, investment opportunities, exogenous spending, monetary policy, price and wage markup shocks. A large set of frictions is introduced so the model-based response of the observed variables to shocks captures some key properties of VAR-estimated IRFs. We describe the main features of the SW model, focusing on the role played by each shock and friction. The reader is referred to the original article for a full description and derivation of the model.

External habit, sticky wages, wage indexation and wage markup shocks. Households maximize a non-separable (GHH) utility function of consumption and labor over an infinite life horizon. Consumption enters the utility function relative to a time-varying external habit variable and labor is differentiated by a union, so there is monopoly power over wages and allows for the introduction of sticky nominal wages as in Calvo (1983). Due to nominal wage stickiness and partial indexation of wages to inflation, real wages adjust only gradually to the desired wage markup. Also, due to time-varying demand elasticity (as in Kimball (1995)), the real wage is a function of expected and past real wages and the exogenous wage markup.

Capital adjustment costs, variable capital utilization and investment-specific technology shocks. Households rent capital services to firms and decide how much capital to accumulate. Capital accumulation is subject to adjustment costs, and capital utilization is variable. The relative efficiency of investment expenditures are subject to investment-specific technology shocks.

Sticky prices, price indexation, TFP and price markup shocks. Firms produce differentiated goods by hiring labor and capital services, set prices as in Calvo (1983) and are subject to shocks to total factor productivity (TFP). Partial indexation of prices and time-varying demand elasticity for differentiated goods are allowed so that current inflation depends on expected future marginal costs, past inflation rate and also on price markup shocks.

Exogenous expenditure, monetary policy, financial frictions and risk-premium shocks. An exogenous expenditure shock is introduced in the model to capture net export revenues or government expenditure shocks. Households can use government bonds to smooth consumption over time. The central bank follows a generalized Taylor rule by gradually adjusting the short-term nominal interest rate of these bonds in response to inflation and output gap (deviation of actual output from the counterfactual flexible-price economy). To capture the degree of interest rate smoothing observed in the US data the Taylor rule is allowed to respond to lagged values according to the autoregressive coefficient ρ_r. Finally, a risk premium shock represents a wedge between the interest rate controlled by the central bank and the return on bonds.

All shocks are assumed to follow an AR(1) process with an IID-Normal error term with zero mean, estimated persistence and standard deviation. Price and wage markup shocks are allowed to incorporate a moving average error term.

5.2 New Features: The Central Bank and The Treasury

The central bank side of the economy is similar to the one developed in section (2). The central bank issues nominal liabilities, buys short and long-term bonds, and make payments to the treasury on a regular basis. The dynamics of the central bank net worth is described by following equation,

$$nw_t = \frac{nw_{t-1}}{\gamma \pi_t} + ni_t - d_t \tag{23}$$

where lower case variables represent detrended real variables. We detrend variables with γ, the steady-state growth rate of the economy, and deflate nominal variables with P_t, the consumer price index. Variables nw_t, ni_t and d_t denote net worth, net income and remittances to the treasury in period t, respectively. The variable π_t denotes the inflation rate between periods t and $t-1$.

As in the model of section (2), we can disaggregate the central bank net income into net interest income and capital gains and losses,

$$ni_t = \underbrace{i_{t-1}\frac{b^{cb}_{t-1}}{\gamma \pi_t} + \left(\frac{1-\delta_b Q^b_t}{Q^b_{t-1}}\right)\frac{b^{l,cb}_{t-1}}{\gamma \pi_t} - i^m_{t-1}\frac{m_{t-1}}{\gamma \pi_t}}_{\text{Net Interest Income } (\equiv nii_t)} + \underbrace{\left(\frac{Q^b_t}{Q^b_{t-1}} - 1\right)\frac{b^{l,cb}_{t-1}}{\gamma \pi_t}}_{\text{Capital Gain}} \tag{24}$$

where b^{cb}_{t-1} and $b^{l,cb}_{t-1}$ denote real value of short and long-term government bonds held by the central bank in period t, respectively. Variable m_t represents the central bank's outstanding monetary liabilities (or reserves), i^m_t is the interest rate paid on reserves (IOR), and i_t is the interest rate paid on short-term government bonds. Q^b_t is the price of long-term government bonds.

In most developed countries, the institutional arrangement between the monetary and fiscal authorities normally determines that a share of the central bank's net income must be remitted to the Treasury. However, the practiced concept of net income differ across countries depending on the type of accounting framework that the central bank adopts. If assets are "marked to maket", net income reflects gains and losses from the variation of long-term bond market prices. However, if a central bank adopts historical pricing in calculating the value of its portfolio, the concept of net income will not incorporate gains and losses from price changes. We incorporate these considerations in the model by considering two types of dividends rules, one based on the net income and another based on the net *interest* income.[13] Moreover, as in the model

[13]This is appropriate in the context of this model because delta bonds are perpetuities that never mature and historical pricing assumes that bonds are worth the nominal principal returned at maturity.

of section (2), we assume that the Treasury does not recapitalize the central bank in case of negative income,

$$d_t = max(0, (1-\zeta)\Theta_t) \quad \text{where } \Theta_t \in \{nii_t, ni_t\} \tag{25}$$

where d_t denotes dividends and ζ is the share of net income (or net interest income) retained at the central bank to build paid-in capital.

Hall and Reis (2015) argue that many central banks have a mechanism that allows them to recover from the issuance of reserves required to make up for negative income. We do that by adding an exclusion clause that authorizes the central bank to refuse to hand over its income to the treasury for a certain period of time. We introduce these considerations in the model by creating a new "deferred assets account". That account gets credited when the Fed's income is negative, and represents a claim on future central bank income, which would have been returned to the treasury according to the dividend rule, but instead is retained at the central bank in order to rebuild the bank's net worth. The deferred assets account is described by the following equation,

$$z_t = \frac{z_{t-1}}{\gamma \pi_t} + (d_t - (1-\zeta)\Theta_t) \tag{26}$$

an the dividend rule gets replaced by,

$$d_t = \begin{cases} 0 & \text{if } \Theta_t < 0 \text{ or } z_t > 0 \\ (1-\zeta)\Theta_t & \text{otherwise} \end{cases} \tag{27}$$

In 2008 the Board of Governors of the Federal Reserve System received authorization to pay interest on balances held by commercial banks at Reserve Banks (IOER). During the monetary policy normalization, the Fed moves the FFR into the target range set by the FOMC primarily by adjusting the IOER. In the context of this model, it is equivalent to setting the interest rate payed on reserves equal to the policy rate,

$$i_t^m = i_t \tag{28}$$

This assumption turns out to be highly convenient in terms of the tractability of the model. It allows us to introduce reserves in the standard SW model without having to make any specific assumptions about the household's demand for liquidity.[14] Moreover, it allows the central bank to tighten monetary conditions without the need to sell assets on its balance sheet. Also, note that, due to the maturity mismatch between the Fed's assets and liabilities, condition (28) does not imply that net interest income will always be equal zero.

We assume that the central bank conducts purchases of short and long-term bonds following

[14] The only assumption underlying $i_t = i_t^m$ is that the money supply must be large enough to satiate the private sector demand for liquidity, which has been a trivial assumption since the start of balance sheet expansion in 2008.

simple autoregressive rules (in terms of detrended real market value),

$$b_t^{l,cb} = (\gamma\pi_*)^{\rho_{cb}} (y_* b_*^{l,cb})^{1-\rho_{cb}} \left(\frac{b_{t-1}^{l,cb}}{\gamma\pi_*}\right)^{\rho_{cb}} exp(\epsilon_t^{l,cb}) \tag{29}$$

$$b_t^{cb} = (\gamma\pi_*)^{\rho_{cb}} (y_* b_*^{cb})^{1-\rho_{cb}} \left(\frac{b_{t-1}^{cb}}{\gamma\pi_*}\right)^{\rho_{cb}} exp(\epsilon_t^{cb}) \tag{30}$$

$$\tag{31}$$

where π_* and y_* denote the steady states of inflation and (detrended) output, respectively, $\rho_{cb} \in (0,1)$ and ($\epsilon_t^{l,cb}$, ϵ_t^{cb}) are i.i.d exogenous shocks. The steady state level of the central bank holdings of short and long-term bonds, b_*^{cb} and $b_*^{l,cb}$, are chosen to match the average size of Fed's balance sheet observed in the ZLB period.[15] In the experiments of the following sections, we feed into the model a sequence of shocks $\epsilon_t^{l,cb}$ and ϵ_t^{cb} so that the model replicates the historical time series of assets held by the Fed. The level of reserves (liabilities) required to back the central bank's purchases of short and long-term bonds is given by,

$$m_t = b_t^{cb,l} + b_t^{cb} - nw_t \tag{32}$$

Finally, as in section (2.4.2), we abstract from government expenditures and assume that the treasury receives dividends from the central bank, d_t, and collects lump-sum taxes, τ_t. The budget constraint of the treasury can be written as,

$$d_t + \tau_t + b_t^{cb} + b_t^{l,cb} + b_t^{hh} + b_t^{l,hh} = (1+i_{t-1})\left(\frac{b_{t-1}^{cb} + b_{t-1}^{hh}}{\gamma\pi_t}\right) + \left(\frac{1+(1-\delta_b)Q_t}{Q_{t-1}}\right)\left(\frac{b_{t-1}^{l,cb} + b_{t-1}^{l,hh}}{\gamma\pi_t}\right)$$

The Treasury adjusts the real primary fiscal surplus in response to the lagged real value of the total government debt, as in Leeper (1991),

$$d_t + \tau_t = exp\left\{\phi_z \left(\frac{b_{t-1}^{hh} + b_{t-1} + b_{t-1}^{hh,l} + b_{t-1}^{l}}{\gamma\pi_t}\right)\right\}$$

where we choose the parameter ϕ_z so that fiscal policy is passive.

The model is log-linearized around its steady-state balanced growth path and cast into a system of linear rational-expectations equations. For later reference, it will be useful to characterize the model in matrix form as,

$$\begin{bmatrix} H_{XX} & 0 \\ H_{xX} & H_{xx} \end{bmatrix} \begin{bmatrix} X_{t+1} \\ \mathbb{E}_t x_{t+1} \end{bmatrix} = \begin{bmatrix} A_{XX} & A_{Xx} \\ A_{xX} & A_{xx} \end{bmatrix} \begin{bmatrix} X_t \\ x_t \end{bmatrix} + \begin{bmatrix} B_X \\ B_x \end{bmatrix} i_t + \begin{bmatrix} C_X \\ C_x \end{bmatrix} \epsilon_t \tag{33}$$

[15]Note that the steady state level of the central bank holdings of short and long-term bonds are expressed as a share of detrended output.)

where X_t is a vector of endogenous predetermined variables, x_t a vector of non-predetermined variables, i_t is the short-term nominal interest rate and ϵ_t is a vector that collects the exogenous shocks. The forward-looking aspect of private agent's behavior is summarized by the lower block of (33). The upper block of (33) is inherited from the past and often describes the dynamic behavior of stock variables.

Solvency Constraints. Let ϕ denote the present value of seignorage revenues. We assume that in each period: (i) the central bank's net worth cannot fall below $-\phi$, and (ii) the balance in the deferred assets account cannot exceed ϕ. The first restriction is similar to section (2), and means that policymakers cannot undertake policy actions that lead to insolvency. According to Hall and Reis (2015), the balance in the deferred assets account is a useful metric for judging the bank's financial stability. A large value of ϕ will prevent a permanent increase of reserves following a negative income shock by cutting subsequent dividends and using the funds to pay off the initial expansion of reserves. However, a balance above ϕ means that the central bank will need to receive a positive transfer from the treasury in present value. We assume that central bankers will dismiss any policy framework that leads to this outcome in order to preserve the independence of the bank. Formally, the solvency constraints are given by the following expressions,

$$nw_t \geq -\phi \quad \text{and} \quad z_t \leq \phi \tag{34}$$

The two restrictions are nearly equivalent when the dividend rule is based on the central bank's net income. However, under the interest-income based dividend rule, the lower bound on the central bank's net worth can be binding while the balance on the deferred account is low. This is because the central bank can be reporting positive net interest income, and hence paying positive dividends to the treasury, while it is suffering large capital losses due to the depreciation of the market value of the bond portfolio.

Optimal Monetary Policy. An advantage of having a structural model based on optimizing behavior is that it provides a natural objective for the monetary policy: the maximization of the expected utility of the representative household. Following the method of Woodford (2003, chap. 6), we can express a second-order Taylor series approximation to this objective as a quadratic function of price inflation, the output gap, and the nominal interest rate.[16] We follow this literature and consider the loss function,

$$L_t \equiv \frac{1}{2}\left[(\pi_t - \pi^*)^2 + \lambda_x \tilde{x}_t^2 + \lambda_i (i_t - i^*)^2\right] = x_t' W x_t$$

where π^* is the inflation target, $\tilde{x}_t$ is the output gap, i^* the steady-state nominal net interest rate, W is a positive definite matrix that collects these variables in the vector of forward looking

[16] Benigno and Woodford (2003) show that we can approximate the policy problem of maximizing the representative household utility by the simpler problem of minimizing a quadratic loss function of inflation and output gap. This approximation assumes that central banks do not care about the path of nonimal interest rate that is required to implement a specific path of inflation and output gap. However, there are substantianl evidence that central banks also seek to reduce the volatility of nominal interest rates (Goodfriend (1991)). Giannoni and Woodford (2003) show that transactions frictions can generate microeconomic-founded justification for interest rate smoothing.

variables, x_t, and λ_x and λ_i are the weights that the central bank attributes to the stabilization of the output gap and interest rate smoothing relative to inflation. We define the intertemporal loss funtion in period t as the expected discounted sum of all the period losses from period t onwards.

Discretion. Here we consider an equilibrium that occurs when policy is conducted under discretion so that the central bank is unable to commit to any future actions. The central bank problem is to choose a sequence $\{i_t\}_{t\geq 0}$ as function of the exogenous process $\{\epsilon_t\}_{t\geq 0}$ and the endogenous state $\{X_t\}_{t\geq 0}$ so as to minimize period-by-period the intertemporal loss function, subject to (33), given a initial condition X_0. The solution of this problem satisfies the following bellman equation,

$$v_t(X_t, \epsilon_t) = \min_{i_t \geq 0} \left\{ \frac{1}{2} x_t' W x_t + \beta \mathbb{E}_t v_{t+1}(X_{t+1}, \epsilon_{t+1}) \right\} \tag{35}$$
$$\text{s.t. (33) and (34) given } X_0$$

Standard methods to find the solution to this problem do not apply in this case because of the large number of endogenous state variables in the system and because of the non-linearity introduced by the ZLB and the solvency constraints. To deal with the non-linearities, we follow Guerrieri and Iacoviello (2015) and solve the model in a piecewise fashion. To deal with the large number of endogenous state variables, we use the optimal linear regulator of a version of the dynamic Stackelberg problem in Ljungqvist and Sargent (2004). We provide a thoroughly description of the solution method in the technical appendix.

Commitment. Here we consider an equilibrium that occurs when policy is conducted under commitment so that the government is able to commit to future actions. Consider minimizing the intertemporal loss function, under commitment once-and-for-all in period $t = 0$, subject to (34), (33) for $t \geq 0$ and X_0 given. The method to find the optimal policy under commitment consists in setting-up the Lagrangian funtion, deriving the first-order conditions, combining these with the model's dynamic equations, and solving the resulting linear rational expectation system using the piecewise linear solution. See the technical appendix for details.

5.3 Generating Forecasts at the Zero Lower Bound.

The model described in the previous section is used to make forecasts of the central bank balance sheet and other key macroeconomic variables in the ZLB period. To generate the forecasts, it is needed to estimate the time series of non-observable variables contained in the vector X_t, and the structural shocks ϵ_t. The standard procedure used in the literature for this estimation is to apply a Kalman filter. One caveat of using this method is that the model with optimal monetary policy is absent from monetary-policy shocks. Hence, to implement the Kalman filter we must exclude from the analysis one of the US data series used to estimate the original model in Smets and Wouters (2007).

To avoid the loss of valuable information, we opted to use an alternative method. As the Kalman filter, it works recursively and requires only the last best guess, rather than the entire history, of the model's state to calculate a new state. It's assumed that the model is at the steady state level immediately prior to 1985:Q1, the first observation of the sample. Given $X_{1984:Q4}$, we choose $\epsilon_{1985:Q1}$ to minimize the model's sum of squared prediction errors (the distance between the model's measurement equations and the observed variables). Calculated the vector of shocks, we can use the model to update the endogenous state, $X_{1985:Q1}$, and repeat the process recursively until 2015:Q4.[17] This method is based on Guerrieri and Iacoviello (2017) and the reader is referred to the article for further details. Section (6) provides a discussion of the estimation results.

5.4 Calibration

The model used to estimate the time series of the non-observed variables, and to make forecasts about the future behavior of the U.S. economy and the Fed's balance sheet, is calibrated separately for the pre-ZLB period (1985:Q1 to 2008:Q3) and the ZLB period (2008:Q4 to 2015:Q4). This division intends to address concerns about the new levels of the natural rate of interest, inflation and output growth, as well as the vast expansion of the size and duration of the Fed's balance sheet since 2008. Table 2 summarizes the calibration.

In the pre-ZLB period, all structural parameters, frictions and shock processes are calibrated equal to the mean of the posterior distribution of the parameters obtained by bayesian methods in Smets and Wouters (2007). In the ZLB period, all paramenters of the pre-ZLB calibration are preserved, except from the intertemporal discount factor, β, the long-run inflation rate, π_*, and the long-run growth trend, γ. These parameters are changed so that the steady-state inflation, GDP growth and nominal interest rate implied by the model are in line with the post-2008 scenario of weak aggregate demand and low growth that has contributed to a general revision of the long-term nominal interest rates in the United States.[18] To recalibrate these parameters we use data based on the FOMC member's individual projections of the nominal interest rate, inflation and output growth under appropriate monetary policy, disclosed by the Summary of Economic Projections (SEP) since 2012:Q1. The mean of the FOMC projections suggest that the new long-run level of the FFR is 4 percent annual, while inflation and GDP growth are 2 percent. Panel BI of table 2 describes the calibration. The parameter π_* is set to 1.005 so that the model's new annual inflation steady state is 2 percent (down from 3 percent in SW07); $\beta = 0.999$ and $\gamma = 1.003$ so that the steady state of the nominal interest rate is 4 percent annual (down from 6% in SW07) and the trend annual growth rate of the economy is 1.2% (down from 1.7% in SW07).

Federal Reserve Bank Balance Sheet. The Fed's balance sheet is calibrated using seven key quarterly time series provided by the Board of Governors of the Federal Reserve System: mon-

[17]The minimization is carried out with an OLS algorithm up to 2008:Q3, the last period before the ZLB binds. In the ZLB period, 2008:Q4 to 2015:Q4, the relationship between the measurement equations and the structural shocks is non-linear because the duration of the ZLB is conditional on the realization of the shock (See Guerrieri and Iacoviello (2015)). In this case, a numerical algorithm is employed to find the vector of shocks that minimize the model's sum of squared prediction error.

[18]Several factors have contributed to generate a long-run downward trend in the equilibrium real interest rate in developed countries and particularly in the United States. Shifts in demographics, a slowdown in trend productivity growth, increase in inequality, the scarcity of safe assets, deleveraging shocks and a reduction in demand for capital goods are the most likely explanations for the decline in the interest rates.See Summers (2013), Taylor (2014), Krugman (2013) and Eggertsson and Mehrotra (2014), Carvalho et al. (2016), Caballero et al. (2016) and Holston et al. (2016).

etary base (Federal Reserve Notes plus deposits held by depository institutions), total capital, assets (U.S. Treasury securities plus mortgage-backed securities), interest income from Treasury securities, interest paid on reserves, dividends on capital stock and earnings remittances to the Treasury (interest on Federal Reserve Notes).[19] We calibrate the steady steady of the model to match the average of each series over the two subperiods, expressed as a share of the quarterly U.S. GDP (QGDP).

Panel A of table 2 summarizes the calibration of the steady state of the central bank's balance sheet and compares it with the averages observed in the data. In the pre-ZLB period, the Fed's bond portfolio was worth on average 22 percent of the QGDP. Most of these assets were backed by liabilities and the Fed's net worth averaged only 0.8 percent of the QGDP during the period. The Fed held only riskless short-term U.S. Treasury bonds and paid no interest on reserves. These operations provided the Fed with an average income stream equal to 0.3 percent of the QGDP, that was almost entirely transfered as dividends to the U.S. Treasury. Column (4) shows that the model and the data line up very well during the pre-ZLB period.

Column (5) of table 2 displays the results for the ZLB period. The main difference between the two periods is the dramatic expansion of the size and duration of the balance sheet. The size of the Fed's assets portfolio averaged 77 percent of the QGDP and the average duration raised to 7.8 years. Since the purchase of these assets was funded essentially by newly created bank reserves, the Fed's liabilities also increased in the period and represented on average 75 percent of the QGDP. The Fed's income and expenses increased proportionately less than the assets and liabilities because the yield curve shifted downwards in the period. Roughly speaking, interest income, dividends, surplus to capital stock and net worth nearly doubled relative to the pre-ZLB era.

As in the endowment-economy model of section (2), we follow Hall and Reis (2015) and set the loss limit, ϕ, equal to the estimated present value of the Fed's seignorage revenues, 8 percent of the QGDP. The depreciation rate of the long-term bond is set to $\delta_b = 0.03$, so that duration equals 7.8 years. Since we assume that the Fed holds only long-term bonds in the ZLB period, the duration of the Fed's assets is also 7.8 years, which is the estimated value-weighted average maturity of the Fed's financial assets between 2009 and 2013 (see Hall and Reis (2015)).[20]

There are other two parameters of our choice to calibrate the central bank's balance sheet: the steady state level of liabilities, m_*, and the share of net income kept at the Fed, ζ. We set $m_* = 0.75$ to match the data. Since there is no evidence that the arrangement between the Fed and the Treasury has changed since 2008, we opted to keep $\zeta = 0.035$, as in the pre ZLB period. Since the observed Fed expenses with interest payments on excess reserves is very small, 0.03 percent of the QGDP, for simplicity we assume $i^m_* = 0$ so that the interest cost is zero in steady state.

Although the implied model's steady state moderately overestimates the Fed's income and net worth observed in the data, it replicates fairly well the main features of the Fed's balance sheet. Note that the model's exaggeration of the Fed's income is due to the fact that we set the steady-state real interest rate at 2% per year, which is too high compared to the actual FFR in

[19] A full description of the data is provided in the Data Appendix.

[20] Hall and Reis (2015) use data in the annual report of the Federal Reserve on the value and maturity of the Treasury securities it holds, to calculate the value-weighted average maturity of the Fed's financial assets

the ZLB period.

Loss Function and the Federal Funds Rate. We follow Giannoni and Woodford (2003) and set the relative weight of output gap to inflation to 1%, $\lambda_x = 0.01$. A positive weight assigned to the stabilization of the nominal interest rate is a necessary condition for stability in the models under commitment and discretion. We choose, $\lambda_i = 0.03$, so that the model-based first-order autocorrelation of the nominal interest rate is in line with the actual FFR autocorrelation estimated over the pre-ZLB sample 1985:Q1 - 2008:Q3.

One caveat of borrowing all parameters governing the shock processes from a model estimated with a Taylor rule, is that the nominal interest rate can display excessive volatility under discretionary optimal monetary policy. We observed that the persistence of the impulse response of the policy rate to a wage mark-up shock is unrealistically high under discretion. As a result of that, the variance of the policy rate largely overestimates the realized variance of the FFR. To restore the good empirical properties of the policy rate, we reduce the persistence of the wage mark up shock. Table (3) shows the actual and model-based autocorrelations and stadandard deviations of the FFR under different calibrations and assumptions of monetary policy. The last column shows that, when we don't adjust the persistence of the wage mark-up shock ($\rho_w = 0.96$), the quarterly standard deviation of the FFR is almost two times larger in the model under discretion than in the data: 13.7% and 7%, respectively. The first column of the table shows that when we set $\rho_w = 0.9$, the standard deviation of the model under discretion reduces to 8.6%, which is slightly higher than the data and, as expected, significantly higher than the model under commitment, 3.5%.

6 Quantitative Model Performance

In this section, we briefly discuss the historical contribution of each structural shock to explain the observed variables over the sample period, and the ability of the model to make reasonable predictions about the federal funds rate and other variables related to the Fed's balance sheet, particularly during the ZLB period.

Shocks and the Great Recession. Figure 5 depicts the estimated structural shocks over the full sample period in the discretion and commitment models. One can see that the risk-premium shock is the most important driver of the great recession. In both models, a very large risk-premium shock hits the U.S. economy in the last quarter of 2007. Following the initial disturbance in the financial sector, a long sequence of adverse investment and expenditure shocks aggravated and lengthened the crisis.[21] Figures 6 and 7 compare the smoothed observed variables with the data. Both models fit well the U.S. time series, especially during the ZLB period.

Federal Funds Rate. We compare the model-based forecasts of the FFR during the ZLB period with the FOMC member's individual projections of the FFR disclosed by the SEP since 2012:Q1. Figures (8) - (11) depict the forecasts of the FFR using the model under commitment (red line), discretion (blue line) and a Taylor rule (green dashed line) with information available in each quarter of the period 2012:Q1 - 2015:Q4. We compare the paths of future interest rate

[21]Note that the model under commitment requires much larger risk-premium shocks to replicate the financial crisis in the last quarter of 2007 than the model under discretion. This is due to the more accommodative monetary policy in the commitment setup.

implied by each model with the mean (red circle), median (green cross) and mode (red cross) with the value that the FOMC's members believe to be the appropriate level of the FFR at selected points in the future (the SEP dots). Black dots display the projections of individual FOMC's members (19 in total). As in Krugman (1998) and Eggertsson and Woodford (2003), the nominal interest rate is kept at the ZLB for a longer length of time under commitment relative to discretion. Figure (12) shows that, while inflation remains persistently below the target when the policy is conducted under discretion or according to a Taylor rule, the model under commitment predicts a quick rebound and overshoot of the 2 percent target. Overshooting the target lowers the real interest rates and stimulates the economy during the ZLB.

Fed's Balance Sheet. Figures 13 and 14 depict actual and model-based Fed's income and expenses under discretion and commitment, respectively. The purple and dashed green lines show the path of smoothed variables from 2005:Q1 to 2015:Q4, when the dividend rule is based on the Fed's net income and net interest income, respectively.[22] One can see that both models track well the upward trend in the Fed's net income during the ZLB period. Because the Fed adopts historical prices in calculating gains or losses in its balance sheet, reported capital gains are zero over the sample period. Note, however, that market evaluations are the main source of volatility in the model-based predictions of income. In a single period, gains and losses from changes in the market price of long-term bonds can be as large as 1.5 percent of quarterly GDP when monetary policy is conducted under discretion. Consequently, the model can replicate satisfactorily the observed stream of dividends when the dividend rule is based on the Fed's net interest income but overestimates the volatility of remittances when the dividend rule is based on the Fed's net income. Also, despite that the Fed has started paying interest on excess reserves since 2008, the total interest cost is relatively small, peaking 0.04 percent of quarterly GDP in 2013-Q4.

We feed the model with a sequence of asset-purchase shocks (see equation (29)) such that total assets held by the central bank in the model replicates the Fed's holdings of U.S. Treasury bonds and mortgage-backed securities observed in the data. We then compare the resulting size of liabilities and net worth that the model generates with those in the data. Figure (15) shows that both models, under discretion and commitment, can capture well the implied increase in the Fed's liabilities as well as the upward trend in net worth during the ZLB period.

Because the Fed holds bonds of different maturities, accurate projections of the Fed's future income and capital losses require a model that captures well not only the dynamics of the FFR but also the behavior of the entire yield curve. Because we make the simplifying assumption that in the ZLB period the Fed holds only bonds with 7.8 years of duration, the observed average duration, it is important that our model is able to make good predictions of this part of the yield curve. Figure (16) display the actual and model-based yields on U.S. Treasury bonds of 1, 2, 5, 10, 20 and 30 years duration. Note the model performs very well with low-yield bonds but the fit deteriorates as the yield increases. However, looking at the yields on 5 and 10-year bonds, the models seems to perform satisfactorily. The model with commitment tracks well the yield trends while the discretion overestimates that by about 25 basis points on average. Noteworthy, both models fail to predict the high volatility of yields in the ZLB period.[23]

[22] For simplicity, we assume that the Fed is fully backed by the Tresury in this execise.

[23] Given the asymmetric arrangement between the Fed and the Treasury, high yield volatility is counterproduc-

Finally, figure (17) shows the smoothed price of long-term bonds under commitment and discretion. When the crisis hit the U.S economy in 2008:Q4, private agents revise their expectation of interest rates downwards and the price of long-term bonds increase substantially: a 14% rise under commitment and 11% under discretion. After the peak, prices converge slowly to their new, and higher, long-run level. However, note that the volatility of convergence is much higher when monetary policy is conducted under discretion, posing a bigger risk to the stability of the Fed.

7 Quantitative Model Results

In this section, we use the quantitative model to test the resilience of the Fed's balance sheet to shifts in the FFR during the normalization of monetary policy following the ZLB period. We then analyze how the solvency constraint forces the Fed to deviate from the baseline optimal policy in order to remain financially sound, and the spillovers on the equilibrium dynamics of inflation and the output gap.

7.1 The Fed's Financial Stability.

We assess the likelihood that the Fed violates at least one of the solvency constraints specified in expression (34) when conducting monetary policy under commitment and discretion during the monetary policy normalization following the ZLB period. We perform Monte-Carlo forecasts with information available in every quarter of the period 2008:Q4 - 2015:Q4, and project the Fed's balance sheet ten years into the future to assess its resilience to the seven types of structural shocks.[24] In order to provide a comprehensive but concise discussion of the main mechanisms driving the dynamics of the Fed's balance sheet, we focus our attention on the announcement date of QE 2 in 2010:Q4. We present a summary of the results for every quarter of the period 2008:Q4 - 2015:Q4 on table (5).

7.1.1 Net-Income Based Dividend Rule.

We start the analysis considering the dividend rule based on the Fed's net income (see equation (27)). Panels (a) and (b) of figure 18 display the paths of key variables simulated with the model under discretion and commitment, respectively. The solid black line shows the smoothed variables until 2010:Q4, shaded gray areas, and the dashed black line represent the percentiles and the median of the forecast distribution, respectively. The red line corresponds to projections in the absence of further shocks to the economy after 2010:Q4.

As expected, monetary policy under discretion is more contractionary than policy under commitment. Looking at the median of the forecast distribution, the Fed keeps the FFR at the ZLB for a longer length of time when operating under commitment. The lift-off takes place

tive for the stability of the bank (high earnings are remitted while high losses are internalized). Hence, if anything, the models underestimate the real risk that monetary policy poses to the Fed.

[24] We estimate the endogenous state of the economy, $X_{2010:Q3}$, and the contemporaneous shock, $\epsilon_{2010:Q4}$, using the method described in section (5.3). Then, we draw 1000 trajectories of the structural shocks from the posterior distribution, solve the model using the piecewise method (described in the technical appendix) and compute the paths of the endogenous variables implied by the model for each realization of the shocks.

in 2013:Q1 under discretion and only 6 quarters later under commitment. Another interesting distinction between the two models is that, while the FFR converges monotonically to the target under discretion, the optimal path of the FFR under commitment first overshoots and only later converges to the target.

Persistence and volatility of the FFR. As in the exogenous-income model, contractionary monetary policy per se does not generate losses to the Fed. The impact of monetary policy on the Fed's balance sheet depends on the market's ability to accurately anticipate the future path of nominal interest rates and incorporate that information in the price of the long-term bond. When the FFR is highly volatile, forecast errors are large and the Fed makes large capital gains and losses due to revaluations of the bond's price, as market participants learn their prediction errors.[25] [26]

Columns (1) and (2) of table (4) report the persistence and forecast error standard deviation from the simulated data of the FFR, long-term bond prices, capital gains and net worth under commitment and discretion, respectively.[27] Because the optimal commitment allows the Fed to smooth the policy-rate response to strutural shocks over time, the implied persistence of the FFR is significantly higher relative to discretion. The estimated autoregressive coeficient of the FFR in the model under commitment is 0.96, implying that an unexpected movement in the FFR has a half life of 16 quarters. In the discretionary setup, the absence of the ability to commit to future policy causes shocks to the FFR to be relatively short-lived, with a half life of only 2.3 periods (autoregressive coefficient equal to 0.74).

When monetary policy is conducted under discretion, the implied FFR is *more* volatile than under commitment. The average standard deviation of the FFR's forecast errors under discretion is 61 basis points, roughly twice as large as the standard deviation implied by the monetary policy under commitment. Figure 18 shows that, under commitment, the lift-off is clustered around 2014:Q3 and there is little dispersion of the FFR in the post-ZLB period. However, under discretion, the duration of the ZLB ranges from 4 to 13 quarters and the distribution of the FFR after the lift-off remains notably volatile. The high volatility of the FFR is passed on to the price of the long-term bond and to the Fed's balance sheet, causing the standard deviation of the Fed's net worth to be more than four times higher under discretion than under commitment, 3.32 and 0.79 percent of quarterly GDP respectively.

The consequences of the high volatility of the FFR are highlighted by both measures of financial strength defined by expression (34). Panel C of table (4) displays the bottom values of the Fed's net worth and the peak values of the balance on the deferred assets account within the 30th, 20th and 10th percentile of the forecast distribution. Column 2 shows that the Fed is in generally sound financial condition under commitment since even in the worst case scenario it does not violate either solvency constraints. The situation is rather different under discretion.

[25] Recall that the equilibrium price of the long-term bond is given by the expected present value of the flow of payments (or coupons) provided by bond.

[26] Note that capital losses are only slightly negative at the median of the FFR distribution, reflecting just the depreciation of the delta bonds and not losses due to unexpected revaluations of the bond price.

[27] We use the simulated data to compute the n-ahead forecast error standard deviation, $\sigma_n \equiv \sqrt{\sum_{i=1}^{m}(x_{i,t+n} - \mathbb{E}_t x_{t+n})^2}$, where $x_{i,t+n}$ is the i^{th} simulated value of variable x in period $t+n$ and m is the number of simulations. The average forecast error standard deviation is the average of σ_n over $n \in \{1, 2, ..., 50\}$. We estimate the persistence of each simulated time path of variable x, $\{x_{i,t}\}_{t=1}^{50}$, from the regression $x_{i,t} = \alpha_i + \rho_i x_{i,t-1} + \epsilon_t^i$. The average persistence of variable x is the average of ρ_i over $i \in 1, ..., m$.

Within the 10th percentile, the Fed's balances on the deferred account reach 8.04 percent of quarterly GDP, which is above the estimated present value of future seignorage revenues, ϕ, violating the second condition of (34). Also in the 10th percentile, the Fed's net worth hits -7.42 of quarterly GDP, coming very close to being technically insolvent and raising serious concerns about the financial stability of the Fed.

Panel A of table (5) reports the quarters in which the Fed violates each solvency constraint within the 5^{th}, 10^{th} and 20^{th} percentiles of the forecast distribution, during 2008:Q4 - 2015:Q4 (between the announcement of QE 2 and the tapering). The results show that the Fed faced a small but positive risk of becoming insolvent. The probability of hitting the constraint on the Fed's net worth is fairly similar to the probability of hitting the upper bound on the deferred account. In summary, while it is highly unlikely that a solvency constraint is violated in 5 of the 13 quarters of the period, there is at least a 5 percent chance that one of the constraints is violated in the other 8 quarters.

7.1.2 Net Interest Income-Based Dividend Rule.

The Fed's financial stability is more vulnerable to interest rate volatility when the dividend rule is based on interest income. Columns (3) and (4) of table (4) show that in 2010:Q4, the Fed violates the solvency constraint even within the 30th percentile of the forecast distribution, under commitment and discretion. Under discretion, the Fed's net worth sinks to -10.9 percent of the quarterly GDP within the 30th percentile of the forecast distribution, and -13.6 within the 10th percentile. Moreover, net worth is much more volatile than the previous case: forecast error standard deviation equals 5.4 percent of the quarterly GDP under discretion and 1.57 under commitment.

Absence of an insurance from the Treasury. When dividends are based on the Fed's net income, remittances to the treasury increase when bond prices are high and is cut back when prices fall. This arrangement benefits the Fed as it shares the risk of holding long-term bonds with the Treasury. This advantage vanishes when the dividend rule does not account for capital gains and losses. Under the net interest income-based dividend rule, the Fed is forced to hand over to the Treasury a share $1 - \zeta$ of the net interest income even if a decline in the price of long-term bonds brings the Fed's net income to negative ground.

Figure (18) shows that interest income becomes a very important source of income as the U.S economy recovers from the crisis and the Fed starts normalizing monetary policy. The growth of net interest income since 2010:Q4 is remarkable, in both models it roughly tripled by 2013:Q4, providing not only a substantial but also stable income flow to the Fed.[28] On the other hand, the Fed suffered large capital losses during the same period: the bond portfolio depreciated on average 0.5% of quarterly GDP per quarter.

Due to the large imbalances between the interest and capital accounts, the overall stability of the Fed depends crucially on the type of dividend rule in place. Figures (18) and (19) show that while the average payout to the Treasury was 0.7 percent of the quarterly GDP per quarter with

[28]Interest income grows despite the fact that the Fed also pays interest on reserves because often the entire yield curve steepens when the Fed raises the FFR (the difference between yields on short-term bonds and yields on long-term bonds increases). Hence, the increment in interest income from the portfolio of long-term bonds outweighs the increase in interest payments on reserves and the Fed's overall net interest income expands

the dividend rule based on the interest income, the Fed sent on average *zero resources* to the Treasury under the net-income dividend rule. As a result of large capital losses and large payouts to the Treasury during an extended period of time, the Fed's net worth deteriorates severely, driving the Fed technically insolvent in more the 50% of the simulated paths, as illustrated in figure (19). The dashed black line shows that the median of the forecast distribution hits the solvency constraint in 2014:Q1.

Panel B of table (5) indicates the quarters in which the Fed violates each solvency constraint during the period 2010:Q4 - 2013:Q4, when the dividend rule is based on the Fed's net interest income. One can see that when conducting monetary policy under discretion, there is at least a 20% probability that the Fed's net worth will fall below -8% of the quarterly GDP *and become technically insolvent*. Moreover, when considering the forecasts in the absence of further shocks to the economy, the Fed violates the net worth constraint in 9 of the 13 quarters. On the other hand, since the payout to the Treasury is almost always positive when the dividend rule is based on the net interest income, the balance on the deferred account is essentially zero throughout the period and the Fed never violates this constraint.

7.2 The Role of the Solvency Constraints.

The previous section concluded that the implementation of QE programs 2 and 3 posed a threat to the Fed's financial soundness, particularly in the case of discretionary policy coupled with a dividend rule based on the Fed's interest income. In this section, we analyze the consequences of imposing the lower bound on the Fed's net worth to the equilibrium dynamics of the FFR, inflation and the output gap.

Figure (20) displays forecasts of the FFR, inflation and the Fed's net worth when the dividend rule is based on the Fed's net interest income. The forecasts are carried out with available information at (*i*) the end of QE 2 in 2011:Q2, (*ii*) the announcement date of QE 3 in 2012:Q3, and (*iii*) Ben Bernake's tapering announcement date in 2013:Q4; and assuming that no further shocks will hit the economy after each forecast initial date (*i*) - (*iii*).[29] The black dashed line shows the predicted evolution of these variables when the monetary authority acts under discretion and is subject to the solvency constraint (34). For comparison, we include the predictions from the baseline discretion (blue line) and commitment (red line) models, in which the solvency constraint is lifted ($\phi \to \infty$).

The top row of the figure (20) illustrates the results for the forecast from 2010:Q4. The baseline discretion model predicts that the FFR will lift off from zero seven quarters later in 2013:Q1, and quickly converge to the 4% targeted rate. This course of action yields insufficient demand and inflation running below the target for several periods. A side effect of this policy plan is the impact on the Fed's balance sheet. The right-hand side plot shows that the rapid

[29] We opted to exclude the QE 1 period from the analysis since the Fed had only moderately extended the maturity of its assets prior to the implementation of QE 2. We also excluded the announcement date of QE 2 because the net worth constraint is not violated when considering the forecasts absent from future shocks (see figure (18) and table (5))

normalization of the FFR drives the Fed's net worth below -8% of the quarterly GDP. The black dashed line shows that to satisfy the solvency constraint, Fed officials must deviate from the baseline policy plan. The liftoff takes place in the same quarter of the baseline discretion but the convergence to the target is slightly slower. In 2014:Q1, the net worth hits - 8 percent of the quarterly GDP and the Fed is forced to cease the tightening cycle for 5 periods. During this interval, a share of the bond portfolio depreciates, making room for further increments of the FFR while preserving the solvency of the Fed.

The impact on inflation is substantial: the average annual inflation rate in the first two years of the forecast increased 0.35 percentage points over the baseline discretion. Figure (21) shows that the impact on the output gap is positive but much weaker than the observed impact on inflation: the accumulated annual output gap within the first two years of forecast increased by only 0.2 percent of quarterly GDP relative to the baseline discretion. This unusual implication of the constrained discretion model can be used to rationalize the common criticism in the literature that DSGE models fail to explain the stabilization of inflation at positive rates in the presence of long-lasting negative output gaps, and others that find a large divergence between the inflation predicted by the baseline Smets and Wouters (2007) model and actual inflation.[30] Finally, the middle and bottom rows of the figure (20) show the results of 2011:Q2 carries over to the announcement date of QE 3 and the tapering.

8 Conclusion

This paper provides a theory of Quantitative Easing. We show that a central bank that is financially independent of the Treasury can use a large-scale purchase of long-term bonds as a commitment device to keep interest rates low in the future. This is because such an open market operation provides an incentive to the central bank to keep interest rates low in future to avoid losses in its balance sheet.

The plausibility of our theory hinges on the assumption that actual QE programs threaten the financial stability of central banks at the exit of the ZLB. We test this hypothesis in a simple stylized model calibrated to the U.S. economy and the Fed's balance sheet and find that although the Fed suffers large capital losses at the exit of the ZLB, it is not subject to insolvency. We then use a workhorse medium-scale DSGE based on Smets and Wouters (2007) and find that the Fed is at risk of insolvency, particularly if monetary policy is conducted under discretion and remittances to the Treasury are based on the Fed's net interest income.

Finally, we use the DSGE model based on Smets and Wouters (2007) to analyze the consequences of the programs QE 2 and QE 3 to the equilibrium dynamics of the FFR, inflation, and the output gap, assuming that the Fed is subject to solvency constraints. We find that the Fed is forced to deviate from the baseline optimal path of the FFR, creating significant additional inflation but mild impacts on the output gap.

[30] See Hall (2011), King and Watson (2012) and Ball and Mazumder (2011).

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9 Figures

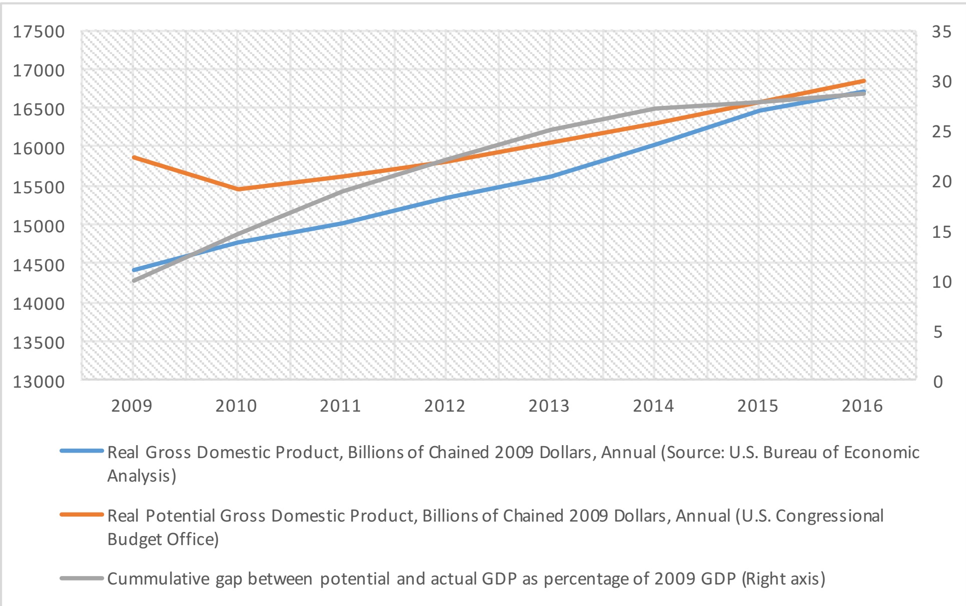
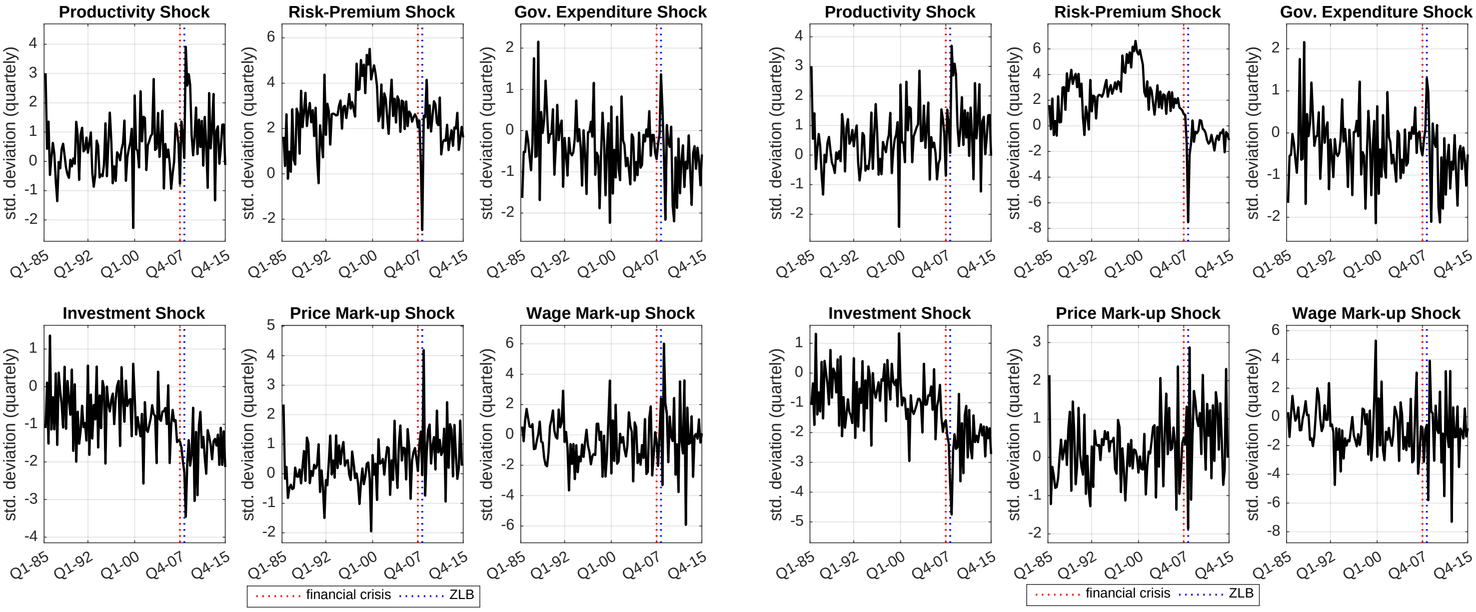


Figure 4: **Benchmark Calibration of the Endowment-Economy Model: Cummulative Gap between Potential and actual GDP as percentage of annual 2009 GDP.**



(a) Discretion

(b) Commitment

Figure 5: **Estimated Structural Shocks from Discretion and Commitment Models (benchmark calibration).** *Notes:* estimation based on the adapted OLS filter in the subsample 1985:Q1 - 2008:Q3 and based on the filter developed in Guerrieri & Iacoviello (2014) in the subsample 2008:Q4 - 2015:Q4.

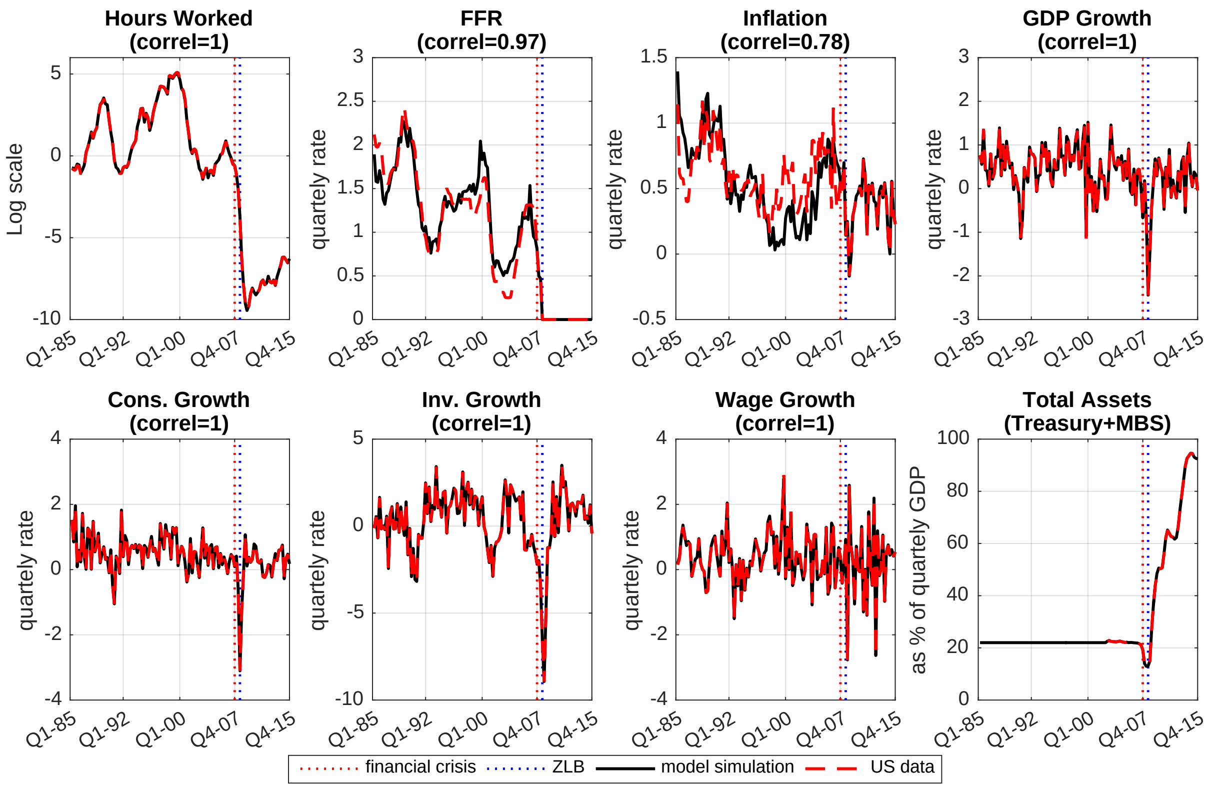


Figure 6: **Actual and Model-Based Observed Variables from the Model under Discretion (baseline calibration).** *Notes:* estimation based on the adapted OLS filter in the subsample 1985:Q1 - 2008:Q3 and based on the filter developed in Guerrieri & Iacoviello (2014) in the subsample 2008:Q4 - 2015:Q4.

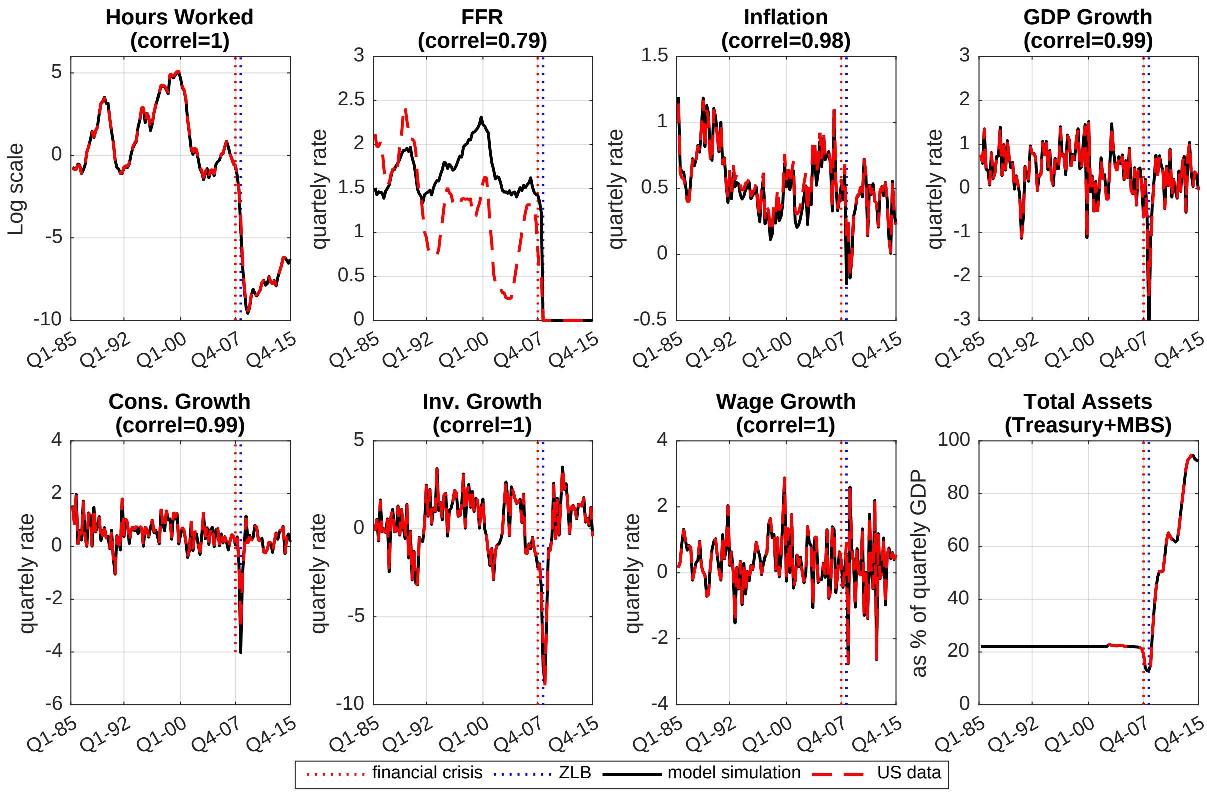


Figure 7: **Actual and Model-Based Observed Variables from Model under Commitment (Baseline calibration).** *Notes:* estimation based on the adapted OLS filter in the subsample 1985:Q1 - 2008:Q3 and based on the filter developed in Guerrieri & Iacoviello (2014) in the subsample 2008:Q4 - 2015:Q4.

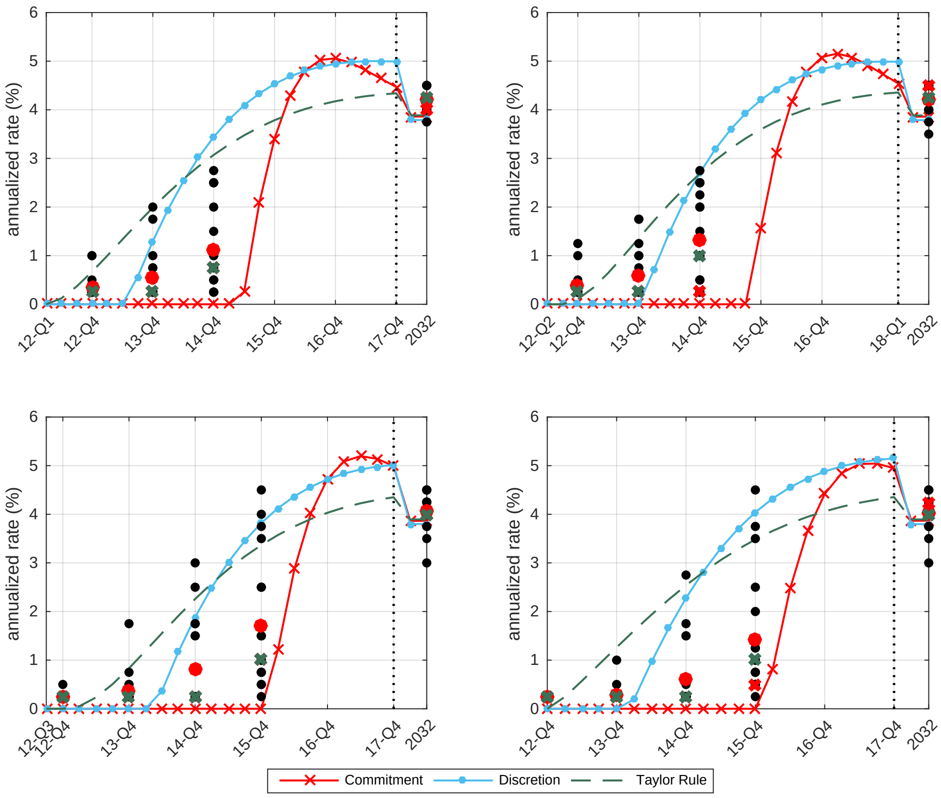


Figure 8: **Forecast of the Federal Funds Rate, 2012: Baseline Calibration of Discretion, Commitment, Taylor Rule and the Summary of Economic Projections.** *Notes:* the red circle, red cross and green cross display, respectively, the mean, mode and median of the FOMC member's individual projections of the Federal Funds Rates under appropriate monetary policy, disclosed by the SEP. Black dots display the projections of individual FOMC's members (19 total).

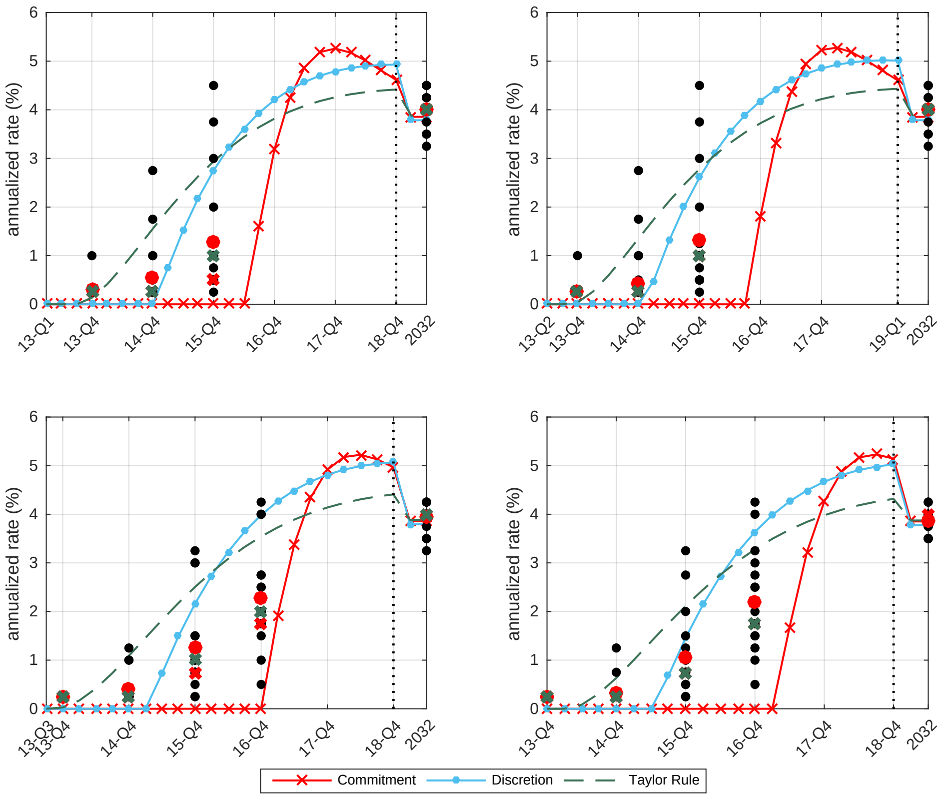


Figure 9: **Forecast of the Federal Funds Rate, 2013: Baseline Calibration of Discretion, Commitment, Taylor Rule and the Summary of Economic Projections.** *Notes:* the red circle, red cross and green cross display, respectively, the mean, mode and median of the FOMC member's individual projections of the Federal Funds Rates under appropriate monetary policy, disclosed by the SEP. Black dots display the projections of individual FOMC's members (19 total).

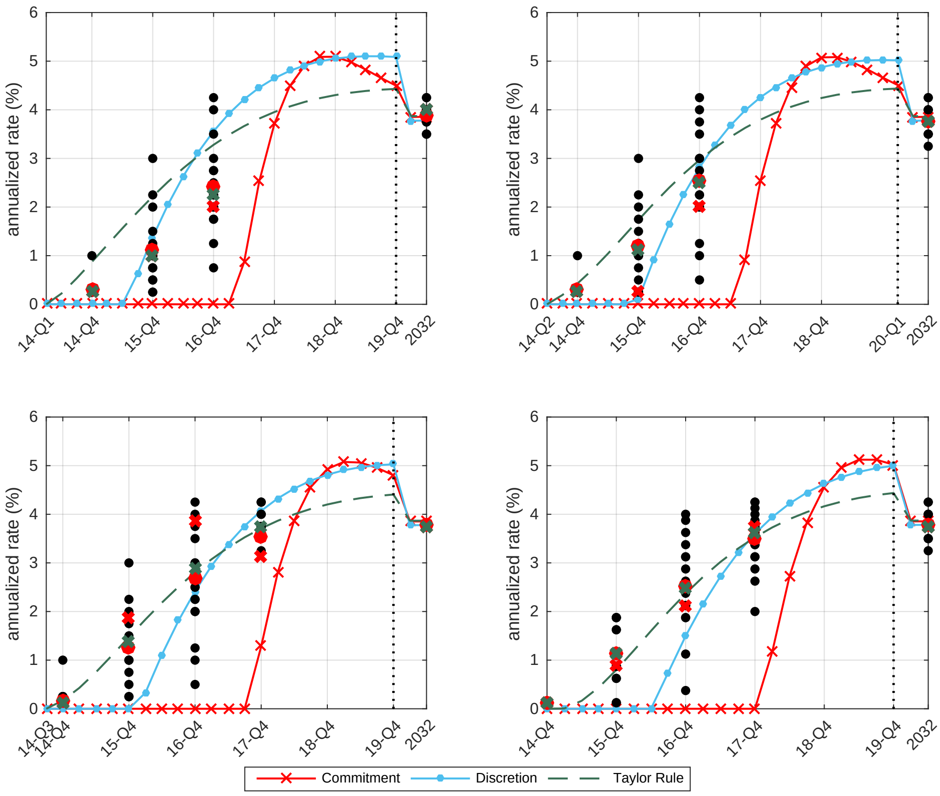


Figure 10: **Forecast of the Federal Funds Rate, 2014: Baseline Calibration of Discretion, Commitment, Taylor Rule and the Summary of Economic Projections.** *Notes:* the red circle, red cross and green cross display, respectively, the mean, mode and median of the FOMC member's individual projections of the Federal Funds Rates under appropriate monetary policy, disclosed by the SEP. Black dots display the projections of individual FOMC's members (19 total).

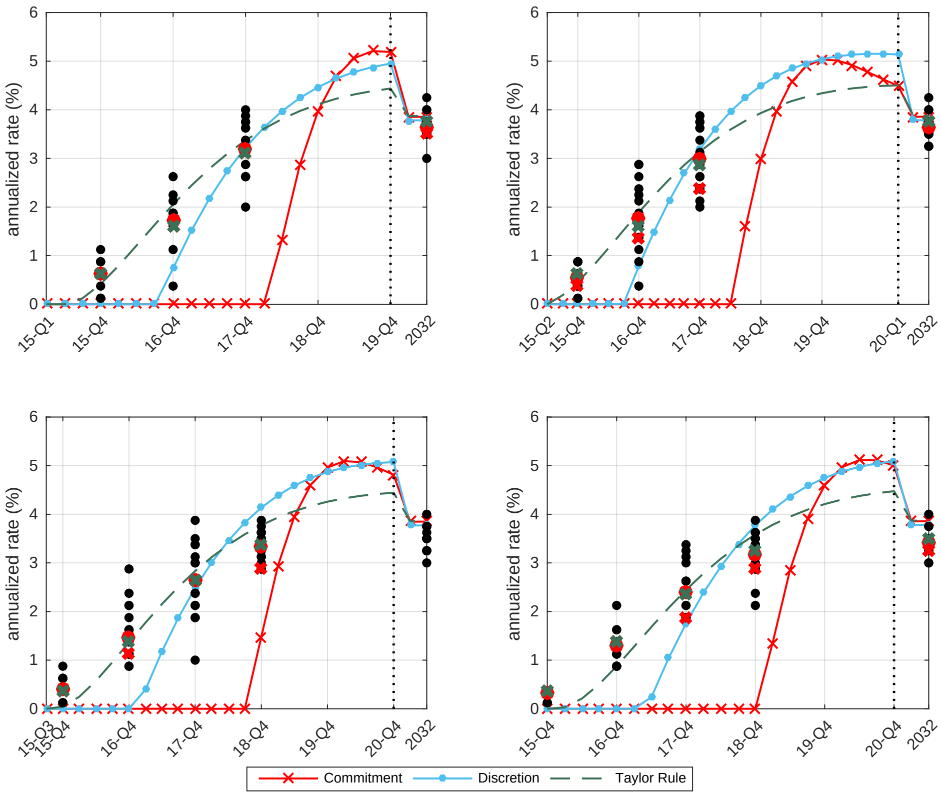


Figure 11: **Forecast of the Federal Funds Rate, 2015: Baseline Calibration of Discretion, Commitment, Taylor Rule and the Summary of Economic Projections.** *Notes:* the red circle, red cross and green cross display, respectively, the mean, mode and median of the FOMC member's individual projections of the Federal Funds Rates under appropriate monetary policy, disclosed by the SEP. Black dots display the projections of individual FOMC's members (19 total).

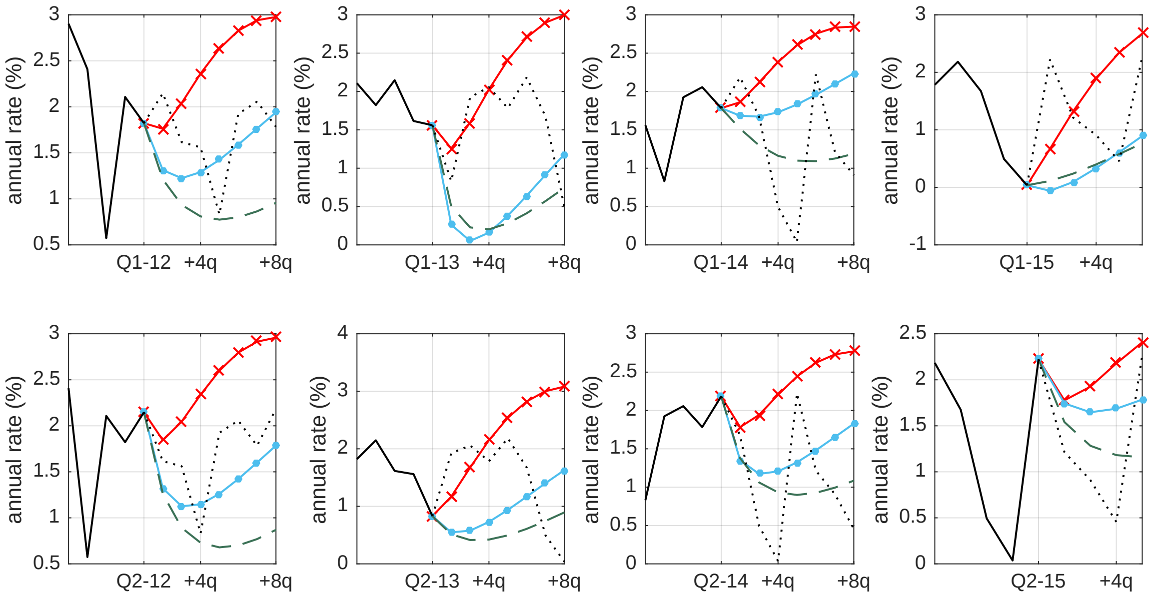


Figure 12: Forecast of the Inflation Rate (PCE) at the Mean of the Distribution, 2012 - 2015: Baseline Calibration of Discretion, Commitment, Taylor Rule and actual US data (black line). First row: projection with information available at the first quarter of 2012 - 2015. Second row: projection with information available at the second quarter of 2012-2015.

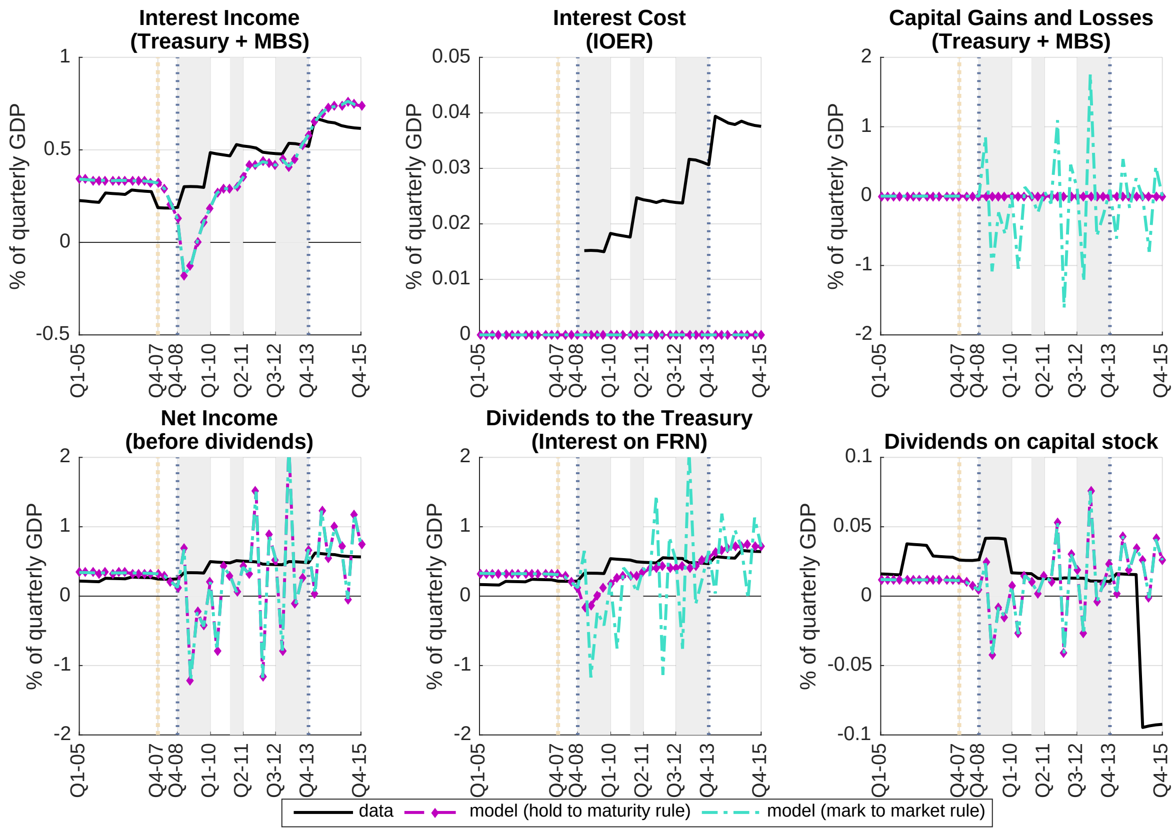


Figure 13: **Actual and Model-Based Fed's Income and Expenses: Smoothed Variables from the Discretion Model (benchmark calibration & full fiscal backing case).** *Notes:* estimation based on the adapted OLS filter in the subsample 1984:Q1 - 2008:Q1 and based on the filter developed in Guerrieri & Iacoviello (2014) in the subsample 2008:Q2 - 2015:Q4. *Data source:* Board of Governors of the Federal Reserve System (US).

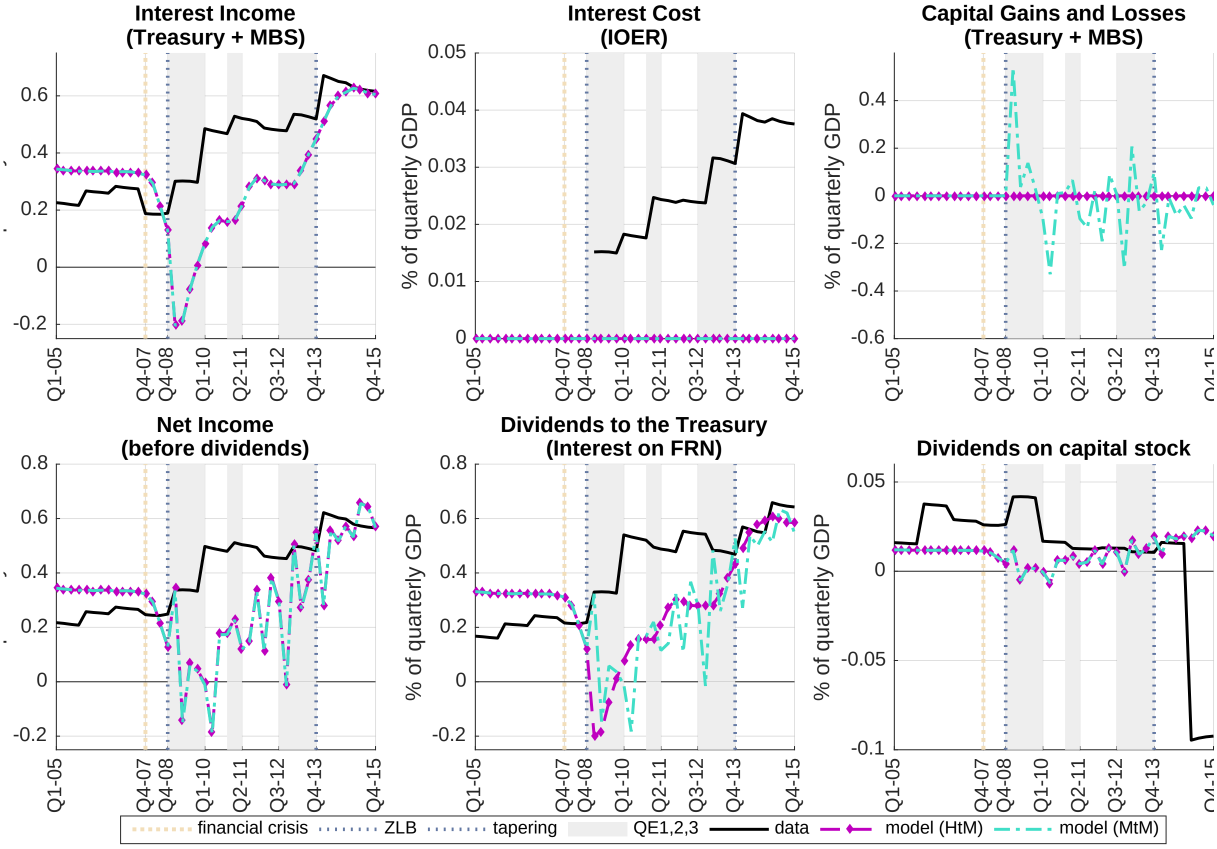


Figure 14: **Actual and Model-Based Fed's Income and Expenses: Smoothed Variables from the Commitment Model (benchmark calibration & full fiscal backing case).** *Notes:* estimation based on the adapted OLS filter in the subsample 1984:Q1 - 2008:Q1 and based on the filter developed in Guerrieri & Iacoviello (2014) in the subsample 2008:Q2 - 2015:Q4. *Data source:* Board of Governors of the Federal Reserve System (US).

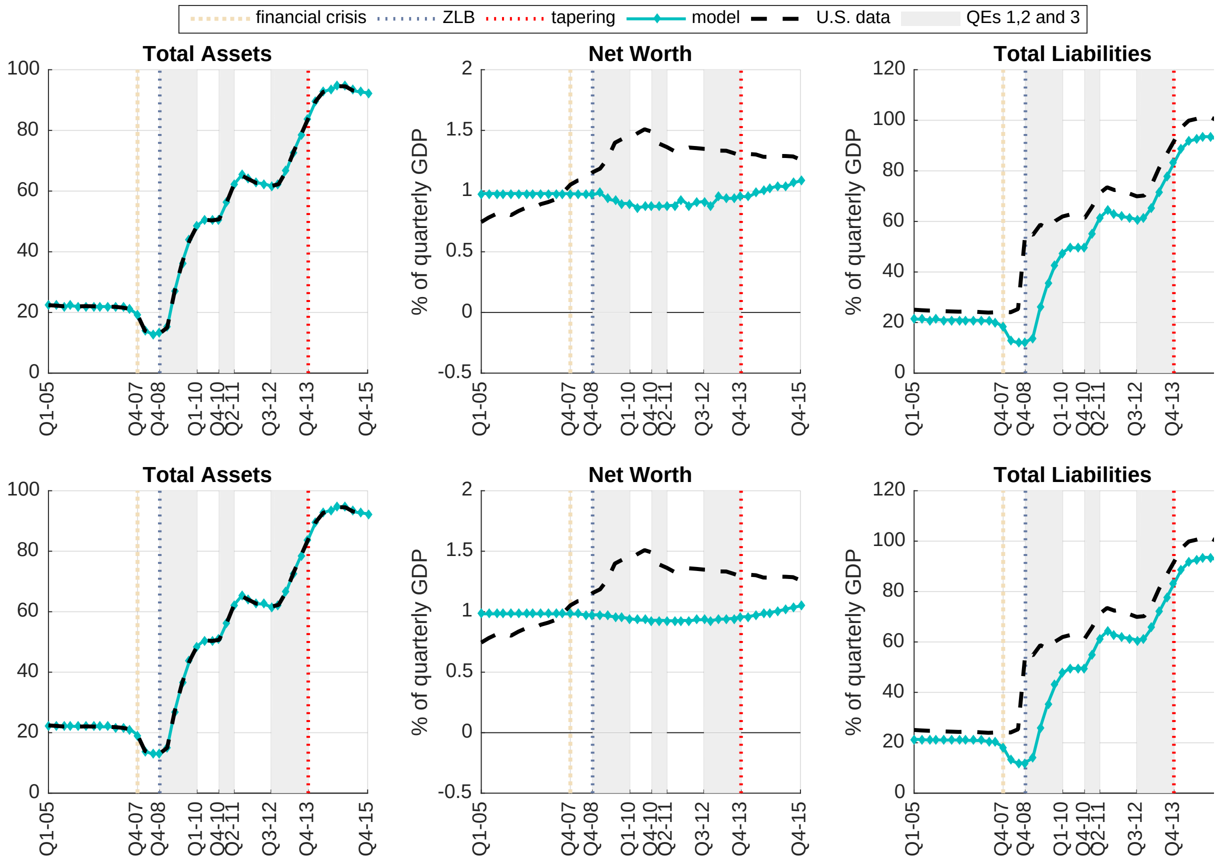
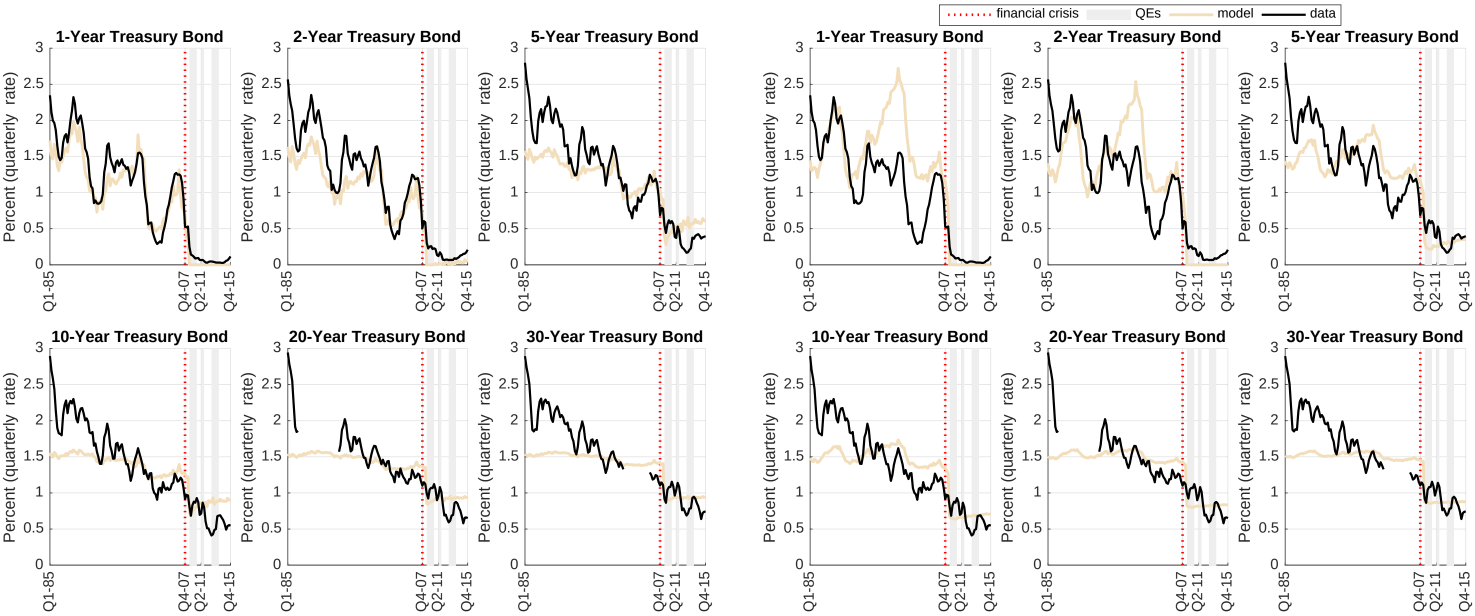


Figure 15: **Actual and Model-Based Fed's Assets and Liabilities: Smoothed Variables from the Discretion (Upper Panel) and Commitment (Lower Panel) Models (baseline calibration & full fiscal backing case).** *Notes:* estimation based on the adapted OLS filter in the subsample 1984:Q1 - 2008:Q1 and based on the filter developed in Guerrieri & Iacoviello (2014) in the subsample 2008:Q2 - 2015:Q4. *Data source:* Board of Governors of the Federal Reserve System (US).



(a) Discretion **(b) Commitment**

Figure 16: **Actual and Model-Based Yields on U.S. Treasury Bonds by Maturities. Smoothed Variables from Discretion and Commitment Models.** *Notes:* estimation based on the adapted OLS filter in the subsample 1984:Q1 - 2008:Q1 and based on the filter developed in Guerrieri & Iacoviello (2014) in the subsample 2008:Q2 - 2015:Q4. *Data source:* Board of Governors of the Federal Reserve System (US).

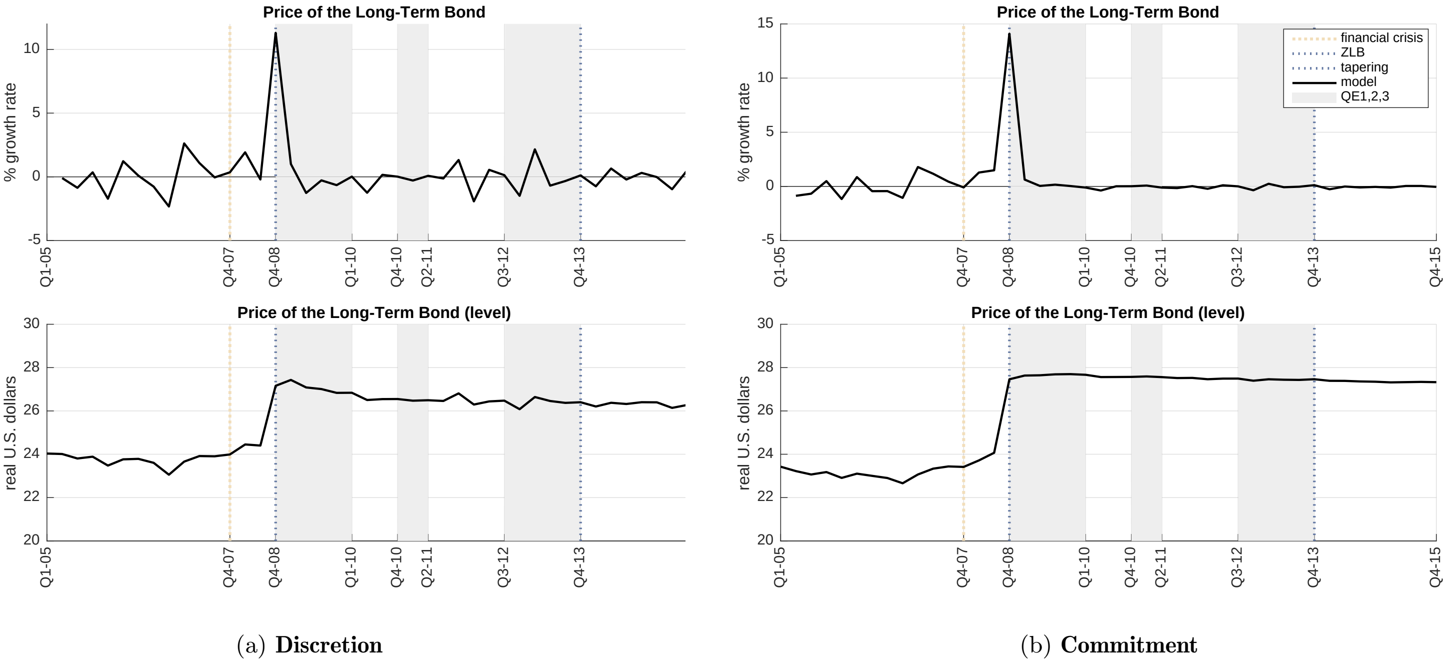
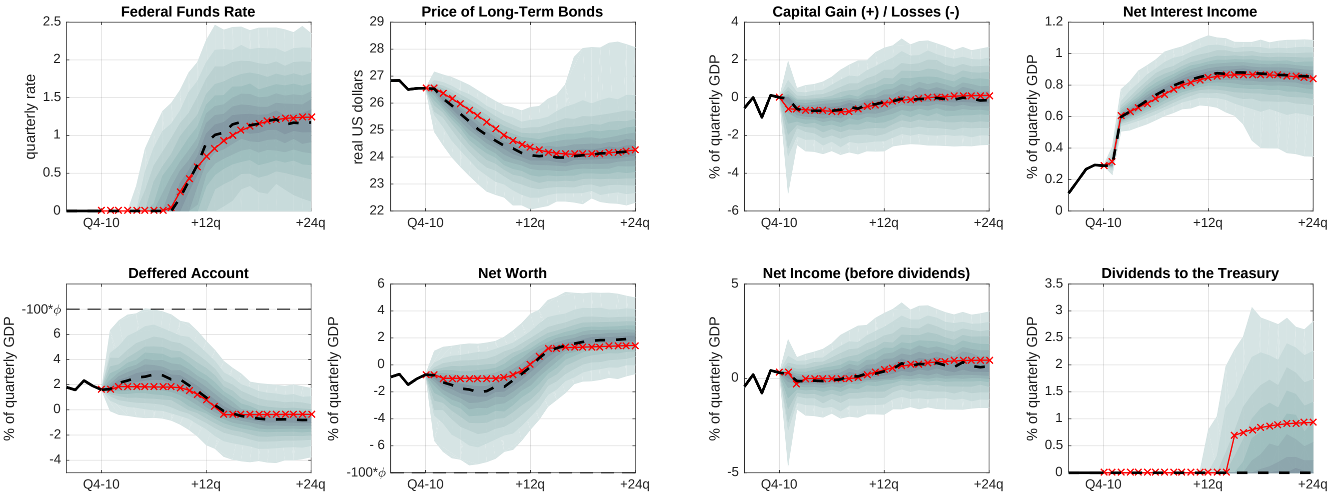
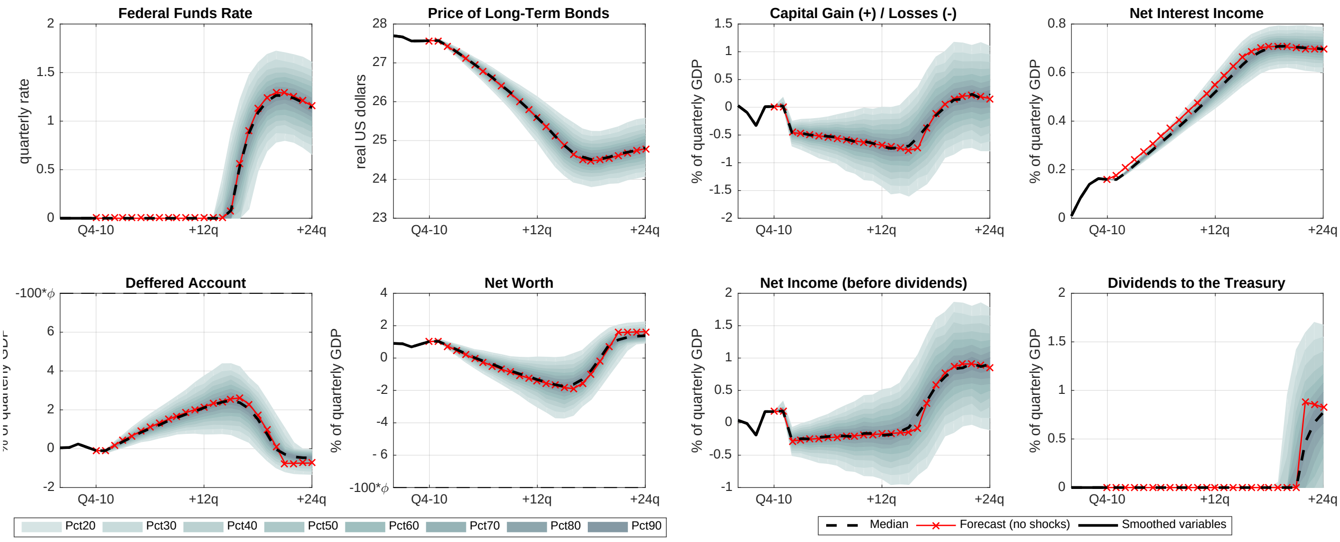


Figure 17: **Smoothed Price of Long-term Government Bonds (7.8-years duration) from Discretion and Commitment Models.** *Notes:* estimation based on the adapted OLS filter in the subsample 1984:Q1 - 2008:Q1 and based on the filter developed in Guerrieri & Iacoviello (2014) in the subsample 2008:Q2 - 2015:Q4.



(a) Discretion



(b) Commitment

Figure 18: **Monte Carlo-Based Forecasts of the Fed's Balance Sheet in the QE 2 Announcement Date: Net Income-Based Dividend Rule & Baseline Calibration.** *Notes:* smoothed variables until 2010:Q4, shaded gray areas and dashed black line represent the percentiles and the median of the forecast distribution, respectively. Red line corresponds to forecasts assuming zero shocks across the forecast period.

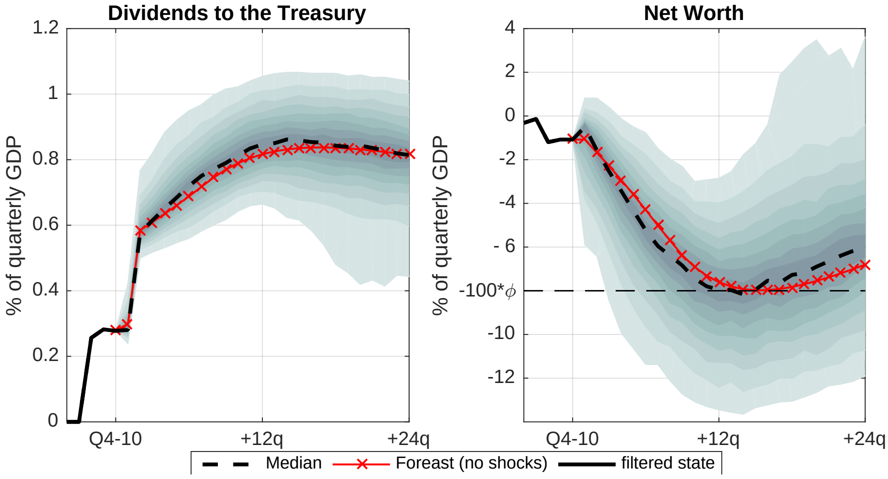


Figure 19: **Monte Carlo-Based Forecasts of the Fed's Net Worth and Dividends in the QE 2 Announcement Date: Discretion & Net Interest Income-Based Dividend Rule (baseline calibration).** *Notes:* smoothed variables until 2010:Q4, shaded gray areas and dashed black line represent the percentiles and the median of the forecast distribution, respectively. Red line corresponds to forecasts assuming zero shocks across the forecast period.

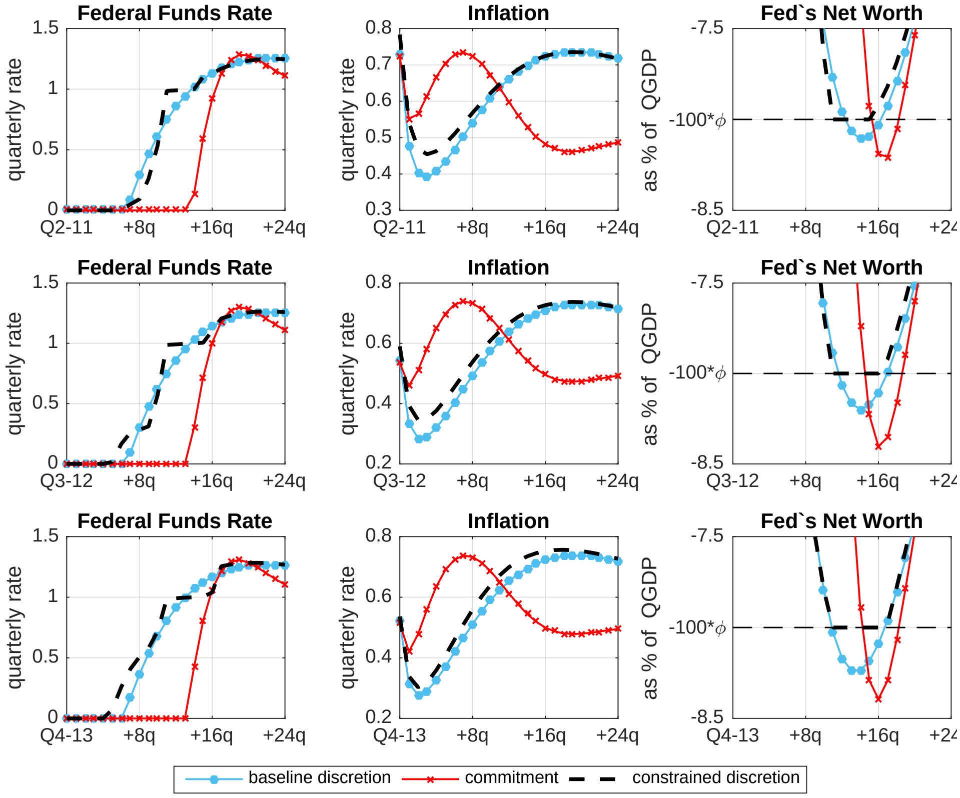


Figure 20: **The Effects of the Solvency Constraint on the Federal Funds Rate and Inflation Dynamics: Baseline Discretion, Commitment and Constrained Discretion.** *Projections with information available at the end of QE 2 (Frist Row - 2011:Q2), announcement date of QE 3 (Second Row - 2012:Q3) and Ben Benanke's tapering announcement date (Third Row - 2013:Q4).*

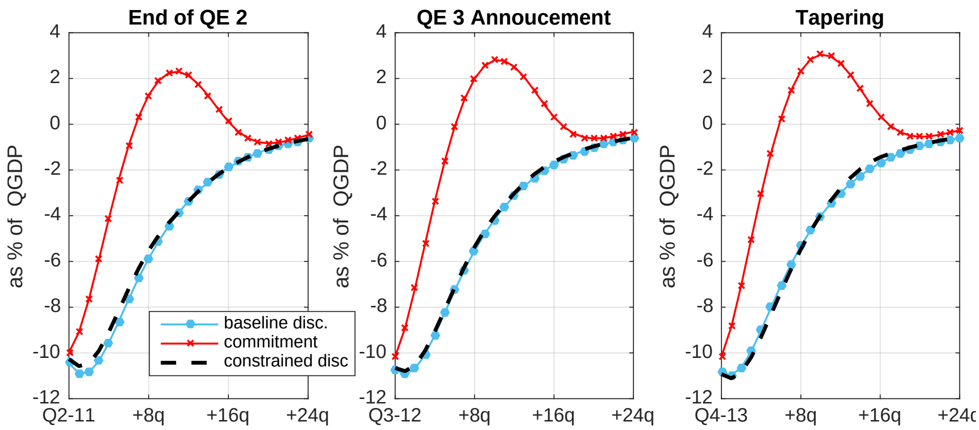


Figure 21: **The Effects of the Solvency Constraint on the Output Gap: Baseline Discretion, Commitment and Constrained Discretion.** *Projections with information available at the end of QE 2 (Frist Row - 2011:Q2), announcement date of QE 3 (Second Row - 2012:Q3) and Ben Benanke's tapering announcement date (Third Row - 2013:Q4).*

10 Tables

Table 1: Endowment Economy Model: Summary of Baseline Calibration and Steady State

	Value	Symbol	Target	Source
Panel A: Parameter Calibration				
Intertemporal discount factor	0.95	β	3-year period & 2% annual real interest rate	
Rate of coupon decay	0.04	δ	7.8 years of Fed's average portfolio maturity	HR2015
Crisis exit probability	0.35	$1-\mu$	5 years of expected crisis duration	HR2015
Pre crisis % income above trend	.035	$\bar{y}$	cumulative loss of output (25% of 2009 GDP)	BEA/CBO
Crisis % income below trend	-.035	$\underline{y}$	cumulative loss of output (25% of 2009 GDP)	BEA/CBO
Loss limit	.007	ϕ	PV of seignorage (2 percent of annual GDP)	HR2015
Coefficient of risk aversion	2	σ		
Panel B: Zero-Inflation Steady State				
Fed's holdings of long-term bonds	0.05	b^l_*	15 pecent of annual GDP	HR2015
Fed's liabilities	0.05	m_*	15 pecent of annual GDP	HR2015
Federal Funds Rate	0.06	i_*	2 percent annual real interest rate	
Interest on Reserves	0.06	i^m_*	2 percent annual real interest rate	
Bond price	5.7	q_*		
Fed's holdings of short-term bonds	0	b_*		
Fed's net worth	0	nw_*		
Fed's net income	0	ni_*		
Remmitances to the Treasury	0	d_*		

Table 2: Quantitative Model: Summary of Baseline Calibration and Steady State: pre ZLB period (1985:Q1 to 2008:Q3) and ZLB period (2008:Q4 to 2015:Q4)

	Value		**Target**	**Model/Data**		**Source**
	(pre ZLB)	**(ZLB)**		**(pre ZLB)**	**(ZLB)**	
	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**
Panel A: Fed Balance Sheet						
AI. Steady State				*as % of Quarterly GDP*		
Liabilities (reserves) (m_*)	0.21	0.75	Avg Fed liabilities	21/21	75/75	FRB/FRED*
Short-term bonds (b_*)	0.22	0	Avg Fed assets	22/22	0/-	FRB/FRED
Long-term bonds (b^l_*)	0	0.75	Avg Fed assets	0/0	78/77	FRB/FRED
Net worth (nw_*)	0.01	0.03	Avg Fed net worth	1/0.8	3/1.3	FRB/FRED
Interest income	.003	.007	Avg Fed int income	0.3/0.3	0.7/0.5	FRB/FRED
Interest cost	0	0	Avg Fed interest cost	0/0	0/0.03	FRB/FRED
Net income (ni_*)	.003	.007	Avg Fed net int inc#	0.3/0.3	0.7/0.4	FRB/FRED
Surplus to capital stock	10^{-4}	10^{-4}	Avg Fed surplus/capital	0.01/0.01	.026/.018	FRB/FRED
Remittances to Treasury (d_*)	.003	.007	Avg Fed remittances	0.3/0.3	0.7/0.4	FRB/FRED
IOER (i^m_*)	0	0	IOER	0/0	0/0.25	FRB/FRED
AII. Parameters						
Rate of coupon decay (δ_b)	0.03	0.03	Avg Fed port/duration	0/<1 yrs	7.8/7.8 yrs	HR 2015
Loss limit (ϕ)	0.08	0.08	PV of seignorage rev	8% of quarterly GDP		HR 2015
Share on paid-in capital (ζ)	.035	.035	Avg Fed net worth/surplus to capital			FRB/FRED
Panel B: Changes relative to SW07						
BI. Steady State (annual %)				*Annual % rate*		
Growth rate trend (γ)	1.004	1.003	SEP FFR/GDP growth	1.7/-	1.2/2	SEP
Inflation rate (π_*)	1.007	1.005	SEP FFR/PCE growth	3/-	2/2	SEP
FFR (i_*)	0.015	0.01	SEP FFR	6/6	4/4	SEP
BII. Parameters				*Annual % rate*		
Intertemporal discount (β)	.998	.999	Federal Funds Rate	6/6	4/4	FRB/SEP**
Weight on stabilization (λ_i)	0.03	0.03	FFR autocorrelation	.86 (disc)	.96 (commit)/0.97	See Table (3)
Persist. wage mk shocks (ρ_w)	0.89	0.89	FFR standard deviation	8.6 (disc)	3.5 (commit)/7	See Table (3)
Weight on output gap (λ_x)	0.01	0.01	Woodford (2003)			

*Board of Governors of the Federal Reserve System (US), retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/TREAST, December 14, 2017.

**Summary of Economic Projections released with the FOMC minutes; https://www.federalreserve.gov/monetarypolicy.htm

#Because the Fed does not report capital gains and losses, net income is equal net interest income.

Table 3: Mordel-Based and Actual Quarterly First-Order Autocorrelation and Standard Deviation of the Federal Funds Rate: Pre-ZLB Sample Period 1985:Q1-2008:Q3.

	Standard Deviation (%) / First-Order Autocorrelation		
	$\rho_w = 0.9$ & $\lambda_i = 0.03$ (*baseline calibration*)	$\rho_w = 0.9$ & $\lambda_i = 0$ (*no FFR stabilization*)	$\rho_w = 0.96$ & $\lambda_i = 0.03$ (*SW07* ρ_w)
Commitment	3.56/0.96	no equilibrium	3.59/0.96
Discretion	8.63/0.86	no equilibrium	13.7/0.91
Taylor Rule	5.17/0.87		5.96/0.90*
Data**	**7.03/0.98**		

* This specification corresponds to the mean of the posterior distribtion in SW07 (sample 1966:Q1 - 2004:Q4).

** Estimates of std. deviation and autocorrelation based on AR(1) reg on FFR time series 1985:Q1-2008:Q4.

Table 4: Statistical Summary of Forecasts of the FFR and the Fed's Balance Sheet at the Announcement Date of QE 2 in 2010:Q4.

Dividend Rule Based on:	*Net Income*		*Net Interest Income*	
	Discretion (1)	*Commitment* (2)	*Discretion* (3)	*Commitent* (4)
Panel A: Forecast error std. deviation.				
Federal funds rate *(basis points)*	61	28	"	"
Price of long-term bond *(real U.S. dollars)*	1.69	0.52	"	"
Capital gains and losses *(as % of QGDP)*	1.44	0.65	"	"
Net worth *(as % of QGDP)*	3.32	0.79	5.44	1.57
Panel B: Persistence of the forecasts				
Federal funds rate	0.74	0.96	"	"
Price of long-term bond	0.83	0.93	"	"
Capital gains and losses	0.01	0.20	"	"
Net worth	0.78	0.93	0.84	0.93
Panel C: Peak/Bottom of the forecasts.				
within the 30th percentile:				
Deferred account	5.01	3.54	-	-
Net worth	-4.29	-2.83	-10.93	-9.25
within the 20th percentile:				
Deferred account *(as % of QGDP)*	5.79	3.77	-	-
Net worth *(as % of QGDP)*	-5.10	-3.07	-11.61	-9.58
within the 10th percentile:				
Deferred account	8.04	4.38	-	-
Net worth	-7.42	-3.71	-13.65	-10.32

* See footnote 27 for the details of the calculation of average persistence and forecast error standard deviation.

Table 5: Fed's Financial Stability under Discretionary Monetary Policy: Summary of the Monte Carlo Forecast Distribution from the Announcement Date of QE 2 in 2010:Q4 to the Annoucement Date of Tapering in 2013:Q4. Affirmative (X) and negative (.)

	2010	*2011*				*2012*				*2013*			
	Q4	*Q1*	*Q2*	*Q3*	*Q4*	*Q1*	*Q2*	*Q3*	*Q4*	*Q1*	*Q2*	*Q3*	*Q4*
Panel A: net-income based dividend rule													
Fed's net worth below -8% of QGDP													
within the 5^{th} percentile	X	X	X	X	.	X	X	.	X	.	.	X	.
within the 10^{th} percentile	.	.	.	.	.	.	.	.	.	.	.	.	.
within the 20^{th} percentile	.	.	.	.	.	.	.	.	.	.	.	.	.
forecast (absence of further shocks)	.	.	.	.	.	.	.	.	.	.	.	.	.
Deferred Account above 8% of QGDP													
within the 5^{th} percentile	X	X	X	X	.	X	X	.	X	.	.	.	X
within the 10^{th} percentile	.	.	X	.	.	.	.	.	.	.	.	.	.
within the 20^{th} percentile	.	.	.	.	.	.	.	.	.	.	.	.	.
forecast (absence of further shocks)	.	.	.	.	.	.	.	.	.	.	.	.	.
Panel B: interest-income based div. rule													
Fed's net worth below -8% of QGDP													
within the 5^{th} percentile	X	X	X	X	X	X	X	X	X	X	X	X	X
within the 10^{th} percentile	X	X	X	X	X	X	X	X	X	X	X	X	X
within the 20^{th} percentile	X	X	X	X	X	X	X	X	X	X	X	X	X
forecast (absence of further shocks)	.	X	X	.	.	X	X	X	X	.	X	X	X
Deferred Account above 8% of QGDP													
within the 5^{th} percentile	.	.	.	.	.	.	.	.	.	.	.	.	.
within the 10^{th} percentile	.	.	.	.	.	.	.	.	.	.	.	.	.
within the 20^{th} percentile	.	.	.	.	.	.	.	.	.	.	.	.	.
forecast (absence of further shocks)	.	.	.	.	.	.	.	.	.	.	.	.	.

Number of simulations = 400.

11 Data Appendix

11.1 Income and Expenses of the Federal Reserve Bank.

- **Total Income**. Board of Governors of the Federal Reserve System. Annual Report, Statistical Tables: Income and Expenses of the Federal Reserve Banks, by Bank. Interest Income/Total current income, Annual data collected year-by-year 1970-2015. https://www.federalreserve.gov/publications/annual-report.htm
  - **Interest Income from Treasury securities**. Board of Governors of the Federal Reserve System. Annual Report, Statistical Tables: Income and Expenses of the Federal Reserve Banks, by Bank. Interest Income/Treasury securities, Annual data collected year-by-year 2005-2015. https://www.federalreserve.gov/publications/annual-report.htm

- **Interest Income from Federal agency and government-sponsored enterprise mortgage-backed securities**. Board of Governors of the Federal Reserve System. Annual Report, Statistical Tables: Income and Expenses of the Federal Reserve Banks, by Bank. Interest Income/Federal agency and government-sponsored enterprise mortgage-backed securities, Annual data collected year-by-year 2005-2015.
  https://www.federalreserve.gov/publications/annual-report.htm
- **Other Income.**. Board of Governors of the Federal Reserve System. Annual Report, Statistical Tables: Income and Expenses of the Federal Reserve Banks, by Bank. Interest Income/Other income, Annual data collected year-by-year 2005-2015.
  https://www.federalreserve.gov/publications/annual-report.htm

- **Net Expenses**. Board of Governors of the Federal Reserve System. Annual Report, Statistical Tables: Income and Expenses of the Federal Reserve Banks, by Bank. Total current expenses/Net expenses, Annual data collected year-by-year 1970-2015.
  - **Interest on Reserves**. Board of Governors of the Federal Reserve System. Annual Report, Statistical Tables: Income and Expenses of the Federal Reserve Banks, by Bank. Current Expenses/Interest on Reserves, Annual data collected year-by-year 2009-2015. https://www.federalreserve.gov/publications/annual-report.htm
  - **Other Expenses**. Board of Governors of the Federal Reserve System. Annual Report, Statistical Tables: Income and Expenses of the Federal Reserve Banks, by Bank. All other expenses, Annual data collected year-by-year.
    https://www.federalreserve.gov/publications/annual-report.htm

- **Net Income**. Total Income - Net Expenses (1970-2015).

- **Capital retained at the Fed** = Surplus + Dividends on Capital Stock (1970-2015)
  - **Surplus retained at the Fed**. Board of Governors of the Federal Reserve System. Annual Report, Statistical Tables: Income and Expenses of the Federal Reserve Banks, by Bank. Distribution of comprehensive income/Transferred to/from surplus and change in accumulated other comprehensive income, Annual data collected year-by-year 1970-2015.
    https://www.federalreserve.gov/publications/annual-report.htm
  - **Dividends on Capital Stock**. Board of Governors of the Federal Reserve System. Annual Report, Statistical Tables: Income and Expenses of the Federal Reserve Banks, by Bank. Distribution of comprehensive income/Dividends on capital stock, Annual data collected year-by-year 1970-2015.
    https://www.federalreserve.gov/publications/annual-report.htm

- **Remittances to the Treasury**. Board of Governors of the Federal Reserve System. Annual Report, Statistical Tables: Income and Expenses of the Federal Reserve Banks, by Bank. Distribution of comprehensive income/Interest on Federal Reserve notes expense remitted to Treasury, Annual data collected year-by-year 1970-2015.
  https://www.federalreserve.gov/publications/annual-report.htm

11.2 Assets, Liabilities and Net Worth of the Federal Reserve Bank.

- **Total Assets.** U.S. Treasury securities held by the Federal Reserve: All Maturities + Mortgage-backed securities held by the Federal Reserve: All Maturities.
  - Board of Governors of the Federal Reserve System (US), U.S. Treasury securities held by the Federal Reserve: All Maturities [TREAST], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/TREAST.
  - Board of Governors of the Federal Reserve System (US), Mortgage-backed securities held by the Federal Reserve: All Maturities [MBST], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/MBST.

- **Total Liabilities.** Board of Governors of the Federal Reserve System (US), Monetary Base; Total [BOGMBASEW], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/BOGMBASEW.

- **Net Worth.** Board of Governors of the Federal Reserve System (US), Capital: Total Capital [WCTCL], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/WCTCL.

11.3 U.S. Treasury Yields and IOER

- **Yields** Board of Governors of the Federal Reserve System (US), 10-Year Treasury Constant Maturity Rate [GS10], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/GS10. *Note: same source for yields on 1, 2, 5, 10, 20 and 30 year Treasury securities.*

- **IOER** Board of Governors of the Federal Reserve System (US), Interest Rate on Excess Reserves [IOER], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/IOER, March 29, 2018.